Endogenous Prices in Markets with Reputational Concerns[*]

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Abstract

I set up a dynamic experience goods model where a long-lived seller interacts with a sequence of short-lived buyers and where the seller has reputational concerns. The key innovation of the article is to endogenize prices in this framework, specifically by endowing either the seller or the buyers with the ability to post price. I show that influence over prices has a strong impact on equilibrium characteristics. If the seller posts price, equilibria will display work-shirk dynamics. If the buyers post price, however, there is always an equilibrium where the seller builds and perpetually maintains her reputation. Buyers always benefit from posting price themselves. Furthermore, sellers may also benefit from buyers posting price.

Keywords: reputation, repeated games, pricing institutions

JEL: C73, D82, L14

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1 Introduction

Markets with information frictions frequently rely on information sharing among market participants to ensure that mutually beneficial transactions can take place. Examples of this reliance are abundant, from rating systems on online trading platforms such as eBay and Amazon, to third-party rating platforms such as Yelp, Tripadvisor, Trustpilot and Google where users can rate a wide range of services. The idea is that market participants can form beliefs about others based on what they have done in the past. The formation of such beliefs may ensure that sellers in a market undertake actions today that go against their immediate self-interest, but benefit the buyers, because acting against one's immediate self-interest may benefit the sellers in the long run.[1] Such beliefs may be referred to as *reputation*. Because the reputation of a market participant may affect future income, reputation can be thought of as an asset. The value of this asset, and the performance of the market, will depend on the price that can be obtained by market participants with a good reputation. Consequently, the way prices are determined may have important implications for the performance of markets with reputational concerns.

This article investigates how influence over price affects outcomes in markets with reputational concerns. I set up a dynamic experience goods model where a long-lived seller (she) interacts with an infinite sequence of short-lived buyers (he). The seller chooses between producing a high- or a low-quality good that is provided in a fixed quantity, and buyers choose between buying the good or not. The seller is either a strategic type who is forward-looking and maximizes payoffs, or a commitment type who always provides high quality, in the spirit of Kreps and Wilson (1982). High quality is costly and the costs depend on the purchasing decision of the buyer. Whereas high quality is preferred by buyers, low quality is always preferred by the strategic seller in the short term. As in Liu (2011) and Liu and Skrzypacz (2014), buyers observe a finite history.[2] Specifically, a buyer entering the game observes the seller´s quality decisions in the most recent $N \geq 1$ periods. The price is endogenized by endowing either the seller or the buyers with full bargaining power, where full bargaining power is modeled as the ability to post price. The posted price is binding and is posted prior to quality and purchasing decisions.

This article makes two main contributions. First, this article introduces strategic price

[1] The empirical relevance of reputational concerns is well established; see Cabral and Li (2015) and Tadelis (2016) for an overview of the empirical literature and Liu et al. (2021) for a more recent contribution.

[2] The assumption that the short-lived side observes only a finite history is reasonable for many markets. For example, most countries with credit bureaus have time limits on reporting information about borrowers and the summary statistic on Amazon covers seller performance over only the last 12 months (Bhaskar and Thomas, 2018).

setting in a model with reputational concerns where buyers are modeled explicitly and where the cost of quality may depend on the purchasing decisions of buyers. This approach differs from the approach in Mailath and Samuelson (2001) and Board and Meyer-ter Vehn (2013) where the purchasing decisions are fixed and where price is determined directly by posterior beliefs.[3] Second, the article considers how influence over price affect equilibrium outcomes in a context with reputational concerns.

The focus of the main analysis is on stationary Perfect Bayesian Equilibria that are constrained efficient. In the following, I refer to these as *efficient equilibria*. I characterize these equilibria with seller-posted prices and with buyer-posted prices and compare them to equilibria of a model where prices are exogenously fixed.

With seller-posted prices, there is a unique efficient equilibrium. In this equilibrium, behavior follows a cyclical pattern where the strategic seller builds reputation for a finite number of periods only to exploit it. Liu (2011) and Liu and Skrzypacz (2014) refer to this as work-shirk dynamics.[4] In the phase of the game where the seller builds reputation, she offers a price equal to the reservation price of the buyers, and always provides high quality, while the buyers mix between purchasing and not purchasing such that the seller is indifferent between high and low quality. In the phase where the seller exploits her reputation, she provides low quality with certainty. In the limit, as the observable history of the seller goes to infinity and the discount factor goes to 1, the seller obtains her Stackelberg payoffs and the equilibrium is efficient.[5] Thus, when the seller posts price, providing buyers with sufficient information (a large N) is key for efficiency.

With buyer-posted prices, there is a continuum of efficient equilibria. However, any efficient equilibrium has the feature that the seller always provides high quality and buyers always purchase on the equilibrium path. In equilibrium, the difference between the price offered to a seller with a history of only high quality and the price offered to a seller with instances of low quality in her history is such that the seller is exactly indifferent between high and low quality. Consequently, the threat of a lower price following a choice of low quality induces the seller to provide high quality. Furthermore, the seller is indifferent, which implies that she can condition her quality choice on the proposed price. Because the threat of low quality following a low-price proposal is credible, the seller earns strictly positive payoffs

[3] I discuss this distinction further in subsection 1.1.

[4] Mohan and Blough (2008) provide evidence for this type of behavior on online trading platforms. Butler (2006) provides similar evidence from the tourism industry.

[5] The Stackelberg payoffs are defined as the payoffs resulting from the long-lived player's preferred pure strategy profile of the stage game under the constraint that the short-run player provides a best response to the long-run player's strategy (Fudenberg and Levine, 1989, 1992).

in equilibrium. As a result, the seller may earn higher payoffs when buyers post price, as compared to when the seller posts price. Finally, note that the equilibria with buyer-posted prices does not require a strictly positive prior on the committed seller and that the equilibria exist for any $N \geq 1$.

While the incentive to build and maintain a reputation partially unravels when the seller posts price, it does not do so when buyers post price. When the seller posts price, the only way buyers can induce high quality is by purchasing at a lower rate after an instance of low quality. However, because the cost of quality is related to the purchasing decision, purchasing at a lower rate following an instance of low quality implies that it is less costly for the seller to rebuild her reputation than it is to maintain it. Furthermore, because the seller has an incentive to rebuild her reputation following an instance of low quality, buyers have incentives to purchase from a seller after an instance of low quality, which they do at a price equal to their reservation price. Thus, the power of dynamic incentives that buyers can provide in an equilibrium with seller-posted prices is limited. With buyer-posted prices, buyers can influence the benefit of building and maintaining a reputation by offering different prices depending on a seller's history while at the same time purchasing at the same rate. Consequently, buyers can provide dynamic incentives without influencing the cost of building and maintaining a reputation.

My results can provide insight into markets where records are limited, either by design as is the case with the summary statistic for sellers on Amazon and credit scores in consumer credit markets, or because complete record keeping is to costly or difficult. I demonstrate that the functioning of such markets may be very sensitive with respects to which side of the market has influence over prices, especially if records are very limited (N is small).[6] Thus, my results have implications for how trade in such markets should be organized. In particular, organizing trade in a way such that buyers have sufficient influence over price may ensure efficiency. In addition, my results have implications for the role of decentralized price setting on platforms that rely on rating mechanism. In particular, my results show that having buyers or the seller strategically set the price, compared to prices being exogenous, may have benefits with respects to markets efficiency. This result speaks against the kind of centralized price setting employed by platforms such as Uber and Lyft as well as the peer-to-peer lending platform Prosper.

[6] As such, my results complement results in Herreiner (2000) and Shi and Delacroix (2018), who ask under which conditions buyers or sellers should organize trade in markets with search frictions. One interpretation of my model is that the side of the market that posts price is the side that organizes trade. Consequently, my work provides some insight into the welfare implications of market organization by showing that markets can benefit from letting the short-lived side of the market organize trade.

1.1 Related literature

My article contributes to two strands of the literature on reputational concerns: reputation effects with limited memories and reputation effects in markets.

Compared to the existing literature on reputation effects with limited memories, my contribution is to study a model with endogenously determined prices. My article is most closely related to Liu (2011) and Liu and Skrzypacz (2014). My article relates to Liu (2011) and Liu and Skrzypacz (2014) in three distinct ways. First, my results qualify the results from these articles by showing that the key insight is not robust to alternative market conditions: Under similar assumptions regarding stage-game payoffs, work-shirk dynamics arise as an equilibrium outcome only when the seller posts price. Thus, my results compliment Levine (2021) who also demonstrate that limited memory is not synonymous with cyclic equilibria.[7] Second, my results also contrast those of Liu (2011) and Liu and Skrzypacz (2014) in a distinct way. In Liu (2011) and Liu and Skrzypacz (2014) equilibria are generally inefficient: To keep buyers indifferent, the seller plays a mixed strategy in the reputation-building phase. This equilibrium resembles the equilibrium in my model when the seller posts price. However, in my model, buyers are kept indifferent through price proposals which enables the seller to provide high quality with certainty in the reputation-building phase and results in deterministic reputation cycles.[8] Consequently, equilibrium outcomes in my model can come arbitrarily close to first-best. Third, in contrast to Liu (2011) and Liu and Skrzypacz (2014), I demonstrate that a long memory length is not a necessary condition for the seller to obtain high payoffs in equilibrium.[9] That is, when buyers post price, the seller can obtain payoffs close to her Stackelberg payoffs regardless of memory length. In that respect, my results compliment Pei (2022b) who demonstrate that the long-lived player can obtain high payoffs when short-lived players have short memories if short lived players cannot observe the exact sequence of historical actions.

My article also contributes to the literature on reputation effects in markets. Compered to existing work in this literature such as Holmström (1999), Mailath and Samuelson (2001) and Board and Meyer-ter Vehn (2013) and Bonatti and Hörner (2017), I study a model where price is modeled as a strategic decision. My article relates most closely to Mailath

[7] Levine (2021) studies a model with one-period memory and with a monitoring structure that implies that signals about the long-lived player´s last action depend also on the actions of the last short-lived player.

[8] Acemoglu and Wolitzky (2014) study a reputation model with limited memory that generates an equilibrium with deterministic cycles. In Acemoglu and Wolitzky (2014) the cycles are driven by random events and not incentives to manipulate beliefs.

[9] A long memory length is also a necessary condition for the long-lived player to obtain high payoffs in Bhaskar and Thomas (2018).

and Samuelson (2001) and Board and Meyer-ter Vehn (2013) who also study a model with a long-lived seller and short-lived buyers. The approach of Mailath and Samuelson (2001) and Board and Meyer-ter Vehn (2013) differs from my approach in two important ways. First, the buyer side is passive and always purchases one unit. Fixing the buyers´purchasing decisions leads to a very different trade-off faced by the seller, as compared to my model, since it implies that the cost of quality depends on the seller´s decision only.[10] Second, price in Mailath and Samuelson (2001) and Board and Meyer-ter Vehn (2013) are is set equal to the expected quality.[11] I demonstrate that allowing price to be a strategic decision rather than fixing it at expected quality, matters for equilibrium outcomes.

The rest of the article is organized as follows: In section 2 I present the baseline model, in section 3 I characterize some general properties of the model as well as equilibria, in section 4 I discus robustness, and in section 5 I conclude.

2 A dynamic quality-choice model with reputation and ultimatum bargaining

In the model, a long-lived seller (S) is matched with an infinite sequence of short-lived buyers (B), one in each period $t \in \{0, 1, ...\}$. In each period, they play a sequential quality-choice game. I study two versions, one where the seller sets the price in each period, and one where the buyers set the price in each period.

2.1 Model

Stage game: In each period either the buyer or the seller first posts a price $p \in \mathbb{R}_+$.[12] The price proposal is then observed by both players and the seller makes a quality choice $q \in \mathcal{Q} = \{0, 1\}$. After the price proposal and the quality choice of the seller, but before observing the quality choice, the buyer chooses whether to purchase $b \in \mathcal{B} = \{0, 1\}$.

High quality ($q = 1$) implies a cost of $c \in (0, 1)$ to the seller if the buyer makes a purchase ($b = 1$). If the buyer chooses not to purchase, the seller incurs no costs.[13] Choosing to buy

[10] The assumptions made in Mailath and Samuelson (2001) and Board and Meyer-ter Vehn (2013) regarding buyer behavior imply that these models do not apply in contexts where the stage game has the type of strategic tensions that arise in, e.g., the Trust game.

[11] Several other articles apply a similar approach, e.g., Dilmé (2019), Jullien and Park (2014), and Bar-Isaac and Deb (2020).

[12] In section 4 I consider the consequence of relaxing this assumptions and letting both sides of the market post a price.

[13] The assumption that the seller carries the cost only if buyers purchase is consistent with Liu (2011) and

implies a transfer from the buyer to the seller of p for a fixed quantity (the quantity is fixed to 1). High quality implies a payoff to buyers, which is normalized to 1 net the price paid. I let $(p, q, b) \in \mathcal{P} \times \mathcal{Q} \times \mathcal{B}$ denote the action profile of any stage game, and let $u_S(q, b) = pb - qbc$ and $u_B(q, b) = (q - p)b$ denote the stage-game payoffs. The seller maximizes her expected discounted payoffs while each buyer maximizes his stage-game payoffs. The seller discounts future payoffs with a discount factor $\delta \in (0, 1)$.

Monitoring structure: I assume that buyers observe past quality decisions only, not past prices or past buyer decisions, and that buyers observe past quality decisions regardless of past buyers´ purchasing decisions.[14,15] Furthermore, like Liu and Skrzypacz (2014), I assume that buyers only observe a limited part of the seller's history. Specifically, buyers can observe $N \geq 1$ previous quality choices of the seller. I let $H = \{0, 1\}^N$ denote the state space of all finite histories with length N. $h = (h_N, h_{N-1}, ..., h_1) \in H$ denotes any realization of H, and h_k denotes the k'th element in h. h_N is the most distant action of the seller in h while h_1 is the most recent. I follow Liu (2011) and define the reputation index $I(h)$

$$I(h) = \begin{cases} N & \text{if } \forall\, k \in \{1, 2, ..., N-1, N\},\ h_k = 1, \\ \min\{k : h_k = 0\} - 1 & \text{otherwise.} \end{cases} \tag{1}$$

$I(h)$ counts the number of instances of high quality since the last instance of low quality and takes on integer values in the range between 0 and N. For example, for a history $h = \{0, 0, ..., 0, 1\}$, $I(h) = 1$; for a history $h' = \{0, 0, ..., 0, 1, 1, 1\}$, $I(h') = 3$; and for a history $h'' = \{0, 1, ..., 1\}$, $I(h'') = N - 1$. Thus, for any given history, one instance of high quality will increase the index from k to $\min\{k + 1, N\}$, and one instance of low quality will reduce the index to 0. In the following, I refer to $I = I(h)$ as the state and $\mathbf{I} := \{0, 1, ..., N - 1, N\}$ as the state space.

Liu and Skrzypacz (2014). In section A of the online appendix, I show that my main results (Propositions 1 and 2) are robust to different cost structures by allowing parts or all of the costs to depend only on the quality choice of the seller.

[14] The assumption that short-lived players observe only the long-lived player´s historical actions, and that they do so regardless of past short-lived players´actions, is common in the literature; see, e.g., Liu (2011), Ekmekci (2011), Liu and Skrzypacz (2014), Heller and Mohlin (2017), and Sperisen (2018). However, there are also many articles where buyers can observe previous buyers' action, see e.g., Fudenberg and Levine (1989), Fudenberg and Levine (1992) and Li and Pei (2021), and where actions of the short-lived player affect the likelihood of the long-lived player´s action being observed by future short-lived players, see Levine (2021). Furthermore, Pei (2022a) demonstrates that buyers´ability to observe previous buyers' action can have a strong effect on equilibrium dynamics in a model with limited memory.

[15] In section 4 I demonstrate that my main results (Propositions 1 and 2) are robust to alternative monitoring structures.

Seller types: There are two seller types: a committed seller and a strategic seller.[16] The seller's type is private information and remains fixed over time. The strategic type maximizes expected payoffs with respect to quality choice (and price proposals if the seller posts price). A committed seller will always provide high quality by assumption, but is fully flexible with respect to price proposals and sets the price so as to *maximize revenue* from sales given buyer beliefs.[17,18] The commonly known prior probability of a committed seller is $\mu > 0$.

2.2 Strategies, beliefs and equilibrium

I assume that buyers do not observe t, but have an improper uniform prior belief $G(t)$ of the period in which they enter the model. Buyers are assumed to update beliefs regarding t according to Bayes's rule. This implies that any buyer in a period $t < N$, that observes at most $N-1$ previous actions, will know t with certainty. Furthermore, any buyer arriving in $t \geq N$ must infer that $t \geq N$, but will be unable to make any further inference. In the main analysis, I focus on periods $t \geq N$ where sellers will have a complete history. In appendix B I characterize equilibrium play in initial periods $t < N$.

In characterizing the equilibria, I consider only strategies which condition only on I (and not H). Furthermore, I restrict my attention to stationary strategies, i.e., strategies that condition only on the present state. The strategy of the side of the market that posts price consists of a price proposal $p : \mathbf{I} \to \mathcal{P}$, and a purchasing decision/quality choice $\sigma_i : (\mathbf{I}, \mathcal{P}) \to [0,1]$.[19] Similarly, let $\sigma_j : (\mathbf{I}, \mathcal{P}) \to [0,1]$, for $j \in \{S, B\}$ and $i \neq j$, denote the strategy of the player who receives the price proposal, where σ_i specifies a probability of high quality or of a purchase. Furthermore, $\boldsymbol{\sigma}_i := \{\sigma_i(I)\}_{I=0}^N$ and $\boldsymbol{p} := \{p(I)\}_{I=0}^N$ denote the sequences of length $N+1$ of stationary strategies that map from I.[20]

Under the assumption of a uniform improper prior $G(t)$ and stationary strategies, the

[16] The presence of a committed seller and the assumptions regarding her behavior are convenient because they simplify the analysis. My main results (Propositions 1 and 2), however, do not rely on these assumptions. In section B of the online appendix, I consider an alternative model similar to that of Pei (2021) where types vary with respect to the cost of quality and demonstrate that there are equilibria similar to those characterized in Propositions 1 and 2.

[17] One interpretation of the committed seller in my model is that it is a seller who has made some initial investment which allows for high quality at no additional cost. I discus the assumptions regarding the committed seller and their implications in section 4.

[18] All further references to *the seller* with regard to strategies and behavior will refer to the strategic seller. When referring to the committed seller, I will be explicit.

[19] Note that I do not consider equilibria where players mix in their price proposals. In the proof of Proposition 1, I demonstrate there cannot be a constrained-efficient equilibrium where the seller mixes with respect to price proposals.

[20] On the equilibrium path, the state maps into a unique price proposal. Consequently, I allow myself the slight abuse of notation by writing σ_i as a function of the state only.

posterior beliefs of buyers upon observing I, but before a price is proposed, are given by Bayes's rule

$$\mu(I) = \begin{cases} \frac{\mu}{\mu+(1-\mu)\lambda(I)} & \text{if } I = N \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

$\lambda(I)$ is the probability of observing state I conditional on the seller being strategic. Thus, $\lambda(I)$ is given by the Markov process induced by $\boldsymbol{\sigma}_S$. If buyers post price, equation 2 is sufficient to measure beliefs. When the seller posts price, however, we also need to condition on the proposed price. Given a price proposal p and a state I, posterior beliefs are given by

$$\phi(I,p) = \frac{\mu(I)Pr(p(I) = p|committed)}{\mu(I)Pr(p(I) = p|committed) + (1-\mu(I))Pr(p(I) = p|strategic)} \tag{3}$$

where $Pr(p(I) = p|committed)$ and $Pr(p(I) = p|strategic)$ are the probabilities (possibly one) that this price is posted by the committed and the strategic seller, respectively. I let $\boldsymbol{\mu} := \{\mu_i(I)\}_{I=0}^N$ denote the sequence of state-dependent beliefs, and $\boldsymbol{\phi} := \{\phi(I,p)\}_{I=0}^N$ denote the sequence of state- and price-dependent beliefs. Note that $\boldsymbol{\phi}$ is relevant only when the seller posts price.

The focus of the analysis is on stationary Perfect Bayesian Equilibria (PBE). In the main analysis (section 3), I restrict my attention to stationary PBE that are constrained efficient. In the following, I refer to stationary PBE satisfying these conditions as an *efficient equilibrium*:

Efficient equilibrium (definition): ($\boldsymbol{\sigma}_S$, $\boldsymbol{\sigma}_B$, $\boldsymbol{p}$, $\boldsymbol{\mu}$, $\boldsymbol{\phi}$) is an efficient equilibrium iff the following conditions are satisfied:

1. **Stationary PBE:** $\boldsymbol{\sigma}_S$, $\boldsymbol{\sigma}_B$, and $\boldsymbol{p}$ are mutual best responses given $\boldsymbol{\mu}$ (and $\boldsymbol{\phi}$), and $\boldsymbol{\mu}$ (and $\boldsymbol{\phi}$) is derived through equation 2 (and 3) whenever possible.

2. **Constrained efficient:** ($\boldsymbol{\sigma}_S$, $\boldsymbol{\sigma}_B$, $\boldsymbol{p}$, $\boldsymbol{\mu}$, $\boldsymbol{\phi}$) satisfies condition 1 and

$$\sum_{I=0}^{N} \lambda(I)[\sigma_S(I)\sigma_B(I) - c\sigma_S(I)\sigma_B(I)] \geq$$
$$\sum_{I=0}^{N} \hat{\lambda}(I)[\hat{\sigma}_S(I)\hat{\sigma}_B(I) - c\hat{\sigma}_S(I)\hat{\sigma}_B(I))]$$

   for all other ($\hat{\boldsymbol{\sigma}}_S$, $\hat{\boldsymbol{\sigma}}_B$, $\hat{\boldsymbol{p}}$, $\hat{\boldsymbol{\mu}}$, $\hat{\boldsymbol{\phi}}$) that also satisfy condition 1.

The goal of the analysis to follow is to characterize all efficient equilibria.

3 Equilibrium analysis

In the following, I characterize the efficient equilibria of the model. Before doing so, I derive some preliminary results regarding the characteristics of stationary PBE of the model. These results simplify the characterization of equilibria and hold regardless of which side of the market posts price.

3.1 Preliminaries

I start by characterizing strategy profiles independently of any particular equilibrium. In doing so, all possible states, $I \in \mathbf{I}$, must be considered. When the seller posts price, and upon observing I, the buyer will choose to buy only if the expected payoff from doing so, given $p(I)$, $\sigma_S(I)$ and $\phi(I,p)$, exceeds that of not buying:

$$\phi(I,p) + \sigma_S(I)(1 - \phi(I,p)) - p(I) \geq 0 \tag{4}$$

Thus, if the expected quality is greater or equal to the price, the buyers' best response will be to purchase. When buyers propose price, we simply replace $\phi(I,p)$ with $\mu(I)$ in equation 4 to obtain the necessary condition for buyers to purchase.

Next, I consider the seller's decision problem and derive the condition under which she will provide high quality. To simplify the notation I define $I^+ = \min\{I+1, N\}$ (I maintain this definition for the rest of the article). In the following, $V_S(I, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \boldsymbol{p})$ denotes the seller's discounted payoffs in some state I, conditional on strategies:

$$\begin{aligned} V_S(I, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \boldsymbol{p}) =& p(I)\sigma_B(I) - \sigma_S(I)\sigma_B(I)c \\ &+ \delta(\sigma_S(I)V_S(I^+, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \boldsymbol{p}) + (1 - \sigma_S(I))V_S(0, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \boldsymbol{p})) \end{aligned}$$

That is, with probability $\sigma_S(I)$, the seller transitions to state I^+ and with probability $(1 - \sigma_S(I))$ she transitions to state 0.

I let $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) := \max_{\boldsymbol{\sigma}_S} V_S(I, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \boldsymbol{p})$ denote the maximized discounted payoffs. That is, $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ are the payoffs resulting from the seller's unilateral best response for a given I, $\boldsymbol{\sigma}_B$ and $\boldsymbol{p}$. Next, let $V_S^1(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ denote the discounted value of the seller's payoffs in any state where she provides high quality. Then $V_S^1(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ is given by

$$V_S^1(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) - \sigma_B(I)c + \delta V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}). \tag{5}$$

Thus, by choosing high quality in state I, the state in the subsequent period will be I^+ and the discounted payoff to the seller in this state is $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})$. The stage-game payoffs are determined by the price times the buyers' propensity to purchase in state I, $(\sigma_B(I)p(I))$, less the costs times the buyers' propensity to purchase $\sigma_B(I)p)$.

Next, let $V_S^0(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ denote the discounted value of the seller's payoffs in any state where she provides low quality. Then $V_S^0(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ is given by

$$V_S^0(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}). \tag{6}$$

. By definition, any instance of low quality will bring the seller to state 0 in the subsequent period, and so the discounted payoffs to the seller in the subsequent period is $V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$. Because low quality has a cost of zero, independently of the buyers' decision, the stage-game payoffs are simply the price times the buyers' propensity to purchase.

For the seller to prefer high quality, the strategies of buyers must be such that $V_S^1(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) \geq V_S^0(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$

$$\sigma_B(I)p(I) - \sigma_B(I)c + \delta V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) \geq \sigma_B(I)p(I) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}).$$

By rearranging, the condition simplifies to

$$\delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \geq \sigma_B(I)c. \tag{7}$$

The condition simply states that the future gain from providing high quality relative to that of providing low quality must be at least as high as the cost of high quality today for the seller to provide high quality.

Equations (4)–(7) are sufficient to prove that the model does not have a stationary PBE where the seller has strict preferences for high quality in any state; see Lemma 1. The proof is contained in appendix A.1.

Lemma 1. *There is no stationary PBE in which* $\delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > \sigma_B(I)c$ *for some* $I \in \mathbf{I}$.

Lemma 1 implies that there cannot exist an equilibrium in which high quality is a strict best response in any state, i.e., if the seller provides high quality in any state in equilibrium, she must be indifferent in that state.

Lemma 1 simplifies the following analysis because it implies that $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ in equation (7) can be replaced with a simpler expression without loss of generality; see Lemma 2. The proof is provided in appendix A.2.

Lemma 2. *In any stationary PBE, and if* $\delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \leq \sigma_B(I)c$ *in all states, then* $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ *and* $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) - \sigma_B(0)p(0)\ \forall\, I \in \mathbf{I}$.

Lemma 2 states that if low quality is a part of a best response, the difference between the discounted payoffs in a state $I > 0$ and state 0 is equal to the difference in stage-game revenues between those states. The intuition is as follows: Independently of the seller's decision in state I, low quality is a best response in the subsequent state (I^+ or 0). Thus, the difference between $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ and $V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ is simply the difference in stage-game payoffs from providing low quality. An immediate implication of Lemmas 1 and 2 is that $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) - \sigma_B(0)p(0)\ \forall\, I \in \mathbf{I}$ in any stationary PBE.[21]

Using Lemma 2, $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ in equation (7) can be replaced with $\sigma_B(I^+)p(I^+) - \sigma_B(0)p(0)$. When doing so, the weak inequality must be replaced with a strict equality. Consequently, the seller is indifferent between high and low quality in state I if

$$\delta(\sigma_B(I^+)p(I^+) - \sigma_B(0)p(0)) = \sigma_B(I)c. \tag{8}$$

For a given price sequence $\boldsymbol{p}$ and $\sigma_B(N)$, equation (8) pins down the sequence $\{\sigma_B(I)\}_{I=0}^{N-1}$.

3.2 Efficient equilibria with a fixed exogenous price

In order to provide a proper benchmark with which to compare the effect of endogenizing prices, I first derive equilibria under the assumption of a fixed and exogenous price, p_F. The detailed analysis and results can be found in appendix C. Here I give only a summary of the main results.

With a fixed price, and if N is sufficiently large, there is a unique efficient equilibrium. In this equilibrium, the seller provides high quality with a probability equal to p_F in all states $I < N$, and low quality in state $I = N$, whereas buyers purchase with a positive probability

[21]The results in Lemmas 1 and 2 are consistent with Liu and Skrzypacz (2014) (see Theorem 1 and its proof). Thus, these equilibrium characteristics are a result of limited records and not endogenous pricing.

in all states $I < N$, and with certainty in state $I = N$. The buyers' strategy is such that the seller is exactly indifferent in all states $I < N$. In the limit, as $N \to \infty$ and $\delta \to 1$, the normalized seller's payoffs converge to her Stackelberg payoffs $(p_F - c)$.

3.3 Efficient equilibria when the seller posts price

In this section I characterize the equilibria of the game described in section 2.1 under the assumption that the seller posts price. When the seller posts price, equilibrium behavior follows a cyclical pattern. In states $I < N$, the strategic seller posts a price of 1 and provides high quality with certainty, while buyers mix such that the seller is exactly indifferent between high and low quality. By assumption, the committed seller will never be in states $I < N$. Consequently, buyers in states $I < N$ know with certainty that they are facing a strategic seller, but buyers still purchase since they know that the seller has incentives to build a reputation. In state $I = N$, which is consistent with both seller types, both seller types post a price equal to the belief on the committed seller, the strategic seller provides low quality with certainty, and buyers purchase with certainty if the proposed price is equal to their belief on the committed seller.[22]

Proposition 1. ***Seller-posted prices:*** *Assume that $\mu > 0$ and $\delta > c$. Then, for any $N \geq 1$, there is a unique efficient equilibrium fully characterized by:*

Price proposals: *$p(N) = \mu(N)$ for both seller types and $p(I) = 1 \; \forall \; I \in \{0, 1, ..., N-1\}$.*

Quality choice: *$\sigma_S(N) = 0$ if $p(N) \leq \phi(p(N), N)$, $\sigma_S(N) = \frac{p(N) - \phi(p(N),N)}{1 - \phi(p(N),N)}$ otherwise, and $\sigma_S(I) = p(I) \; \forall \; I \in \{0, 1, ..., N-1\}$.*

Beliefs: *$\mu(I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$ and $\mu(N) = \frac{\mu(N+1)}{\mu N + 1}$; $\phi(p(I), I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$, $\phi(p(N), N) = \mu(N)$ if $p(N) = \mu(N)$ and $\phi(p(N), N) \leq \mu(N)$ otherwise.*

Purchasing decision: *$\sigma_B(N) = 1$ if $p(N) \leq \phi(p(N), N)$ and $\sigma_B(N) = \sigma_B(N-1)$ otherwise, $\sigma_B(I) = \frac{\delta(p(I^+)\sigma_B(I^+) - \sigma_B(0))}{c} \; \forall \; I \in \{1, 2, ..., N-1\}$ if $p(I) \leq 1$ and $\sigma_B(I) = 0$ otherwise, and*

$$\sigma_B(0) = \begin{cases} \frac{p(N)}{k(N+1)} & \text{if } p(0) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where $k(x) = \frac{\delta}{\delta - c}\left(1 - \left(\frac{c}{\delta}\right)^x\right)$.

[22] Proposition 1 does not mention the case in which $\mu = 0$. In this case, $\mu(I) = 0$ for all states. As such, there is no incentive for the seller ever to provide high quality, and there cannot be an equilibrium where she provides high quality. See Liu (2011) or Bhaskar and Thomas (2018) for a formal argument.

The proof of Proposition 1 is contained in appendix A.3.[23] Below I discuss some specific aspects of the result.

First, consider the equilibrium price proposals in states $I < N$. The seller posts a price of 1 in any state $I < N$. To see why this must be the case, consider the following. In an equilibrium, buyers must have correct beliefs regarding the price profile $\boldsymbol{p}$. The belief about $\boldsymbol{p}$ and play in state $I = N$ uniquely determine the sequence $\boldsymbol{\sigma}_B$ such that the seller is indifferent. Thus, for any candidate equilibrium in which $p(I) < 1$ in some state $I < N$, the seller would have a strict incentive to deviate and post a price of 1 instead as this would not affect the buyer´s propensity to purchase. For some intuition, let $N = 1$ and fix a candidate equilibrium in which $p(0) = p^0 < 1$ while everything else is as in Proposition 1. Then, by equation (8), $\sigma_B(0)$ is pined down by $\delta(p(1) - \sigma_B(0)p^0) = \sigma_B(0)c$. Denote this mix by σ_B^0. Note that p^0, which together with $p(1)$ determines σ_B^0, is the price that buyers believe that the seller will post in state $I = 0$ and that buyers also believe that future buyers in state $I = 0$ will purchase at a rate of σ_B^0. Next, let $p_t(0)$ denote the price the seller posts in a period t in which the state is $I = 0$. If the seller deviates and posts a price of $p_t(0) = 1$ instead of $p_t(0) = p^0$, the LHS of equation (8) will remain the same as it is determined by future play and not the current price proposal. That is, under the assumption of the equilibrium candidate, buyer beliefs regarding future play are fixed regardless of any off-path price proposal. Thus, the buyer in period t will still have to mix according to σ_B^0 because any other mix will result in the seller either having strict preferences for high quality ($\delta(p(1) - \sigma_B^0 p^0) > \sigma_B(0)c$) or strict preferences for low quality ($\delta(p(1) - \sigma_B^0 p^0) < \sigma_B(0)c$). Consequently, the seller strictly prefers to deviate and post $p_t(0) = 1$, rather than $p_t(0) = p^0$. As the seller would prefer to deviate from $p_t(0) = p^0$, the candidate does not constitute an equilibrium.

Second, note that the endogenous price formation induced by seller-posted prices does not mitigate work-shirk dynamics. Because the seller will post a price of 1 in all states $I < N$, any incentives to provide high quality in state $I = N$ will have to come from a sufficiently severe punishment from buyers in state $I = 0$. However, buyers cannot commit to a sufficiently severe punishment in state $I = 0$. To see this, consider again the case in which buyers have a one-period memory ($N = 1$). Let $\sigma_B'(0)$ denote the mix in state $I = 0$ that would make the seller indifferent in $I = 1$. For the seller to be indifferent in $I = 1$ the following condition

[23] In section D of the online appendix I demonstrate that the equilibrium in Proposition 1 is purifiable, in the sense of Harsanyi (1973), but only consider the special case of $N = 1$.

would have to hold:

$$\delta(p(1) - \sigma'_B(0)) = c \Leftrightarrow \sigma'_B(0) = \frac{\delta p(1) - c}{\delta}.$$

Next, from equation (9), the buyers in state $I = 0$ must mix according to $\sigma_B(0) = \frac{\delta p(1)}{\delta + c}$. Clearly, $\frac{\delta p(N)}{\delta + c} > \sigma'_B(0) = \frac{\delta p(1) - c}{\delta}$. Thus, if buyers were to play according to $\sigma_B(0) = \sigma'_B(0)$, the seller would strictly prefer high quality in state $I = 0$. Furthermore, if the seller has a strict preference for high quality, buyers must also strictly prefer to purchase. As such, $\sigma'_B(0)$ cannot be an equilibrium strategy. Intuitively, while a more severe punishment (a lower $\sigma_B(0)$) will increase the incentive to provide high quality in any state, it will also lower the cost of building reputation in state $I = 0$, which further increases the incentive of the seller to provide high quality in that state. Thus, any $\sigma_B(0)$ that makes the seller indifferent in state $I = N$ will imply that the seller strictly prefers high quality in state $I = 0$.

Third, the equilibrium has the property that $\sigma_B(I+1) > \sigma_B(I)$ for all $I \in \{0, 1, ..., N-1\}$. Thus, Proposition 1 shows that the reputation-bubble dynamics described in Liu (2011) and Liu and Skrzypacz (2014) are robust to endogenous prices when the seller posts price. Also, in relation to this property, and as Liu and Skrzypacz (2014) point out, in a model such as this, where $\mu(I) = 0$ for $I < N$, the posterior belief is not a sufficient statistic to characterize equilibrium behavior. If $I^+ < N$, buyers in both state I and state I^+ will believe with certainty that they are facing a strategic seller. Thus, the index I is critical for characterizing equilibrium play because it alone informs buyers on how they must play to make the seller indifferent.

Fourth, the equilibrium relies on the specification of off-path beliefs for any $p(N) \neq \mu(N)$. Off-path beliefs are not very restricted, because they imply only that any deviation from the equilibrium price proposal does not increase the posterior belief in the committed seller.[24] Note also that the set of permissible off-path beliefs includes $\phi(p(N), N) = \mu(N)$ for all $p(N)$. This off-path belief is consistent in the sense that if I perturb the price proposal strategy of both seller types by adding a slight tremble, $\phi(N, p(N)) = \mu(N)$ for any $p(N)$ follows directly from equation 3. Note also that such a tremble, and its implication for off-path beliefs, would eliminate any equilibrium with prices $p(N) < \mu(N)$ because both seller types could then profitably deviate by posting a price equal to $\mu(N)$.[25]

A final thing to notice, that distinguishes the equilibrium characteristics of my model

[24] This is without loss of generality: In the proof of Proposition 1, I show that there cannot exist equilibria in which $\phi(p(N), N) > \mu(N)$.

[25] Equilibria with prices $p(N) < \mu(N)$ are not considered, because they are not constrained efficient.

from that of Liu and Skrzypacz (2014) (and the version of the model with a fixed price), is that the seller provides high quality with certainty in the reputation-building states ($I < N$). In Liu and Skrzypacz (2014), the seller plays a mixed strategy that ensures that buyers are indifferent. The price, exogenously fixed or endogenously determined, will work as a commitment device by implying a lower bound on quality whenever buyers purchase in equilibrium. However, because buyers must be indifferent in equilibrium, the price also implies an upper bound. Consequently, if the price is exogenously bounded below 1, the seller cannot provide high quality with certainty in equilibrium. By allowing the price to be determined endogenously, however, the seller can keep buyers indifferent through price proposals and provide high quality with certainty in states $I < N$.[26]

Thus, while endogenous pricing does not mitigate work-shirk dynamics, it induces the seller to provide high quality at a higher rate in the reputation-building phase. As the following corollary formalizes, this has implications for the limit properties of the efficient equilibrium as $N \to \infty$. The proof is located in appendix A.4.

Corollary 1. ***Convergence to Stackelberg payoffs**: When the seller posts price, normalized payoffs in the efficient equilibrium and in any state $N - i$ converge to $1 - c$ as $N \longrightarrow \infty$ for $i \in \{1, 2, ..., N-2, N-1\}$. Furthermore, normalized payoffs in the efficient equilibrium in state $I = 0$ converge to $1 - c$ as $N \longrightarrow \infty$ and $\delta \longrightarrow 1$.*

The intuition is as follows: When N increases, the price in state $I = N$ will increase as well. Thus, to keep the seller indifferent in states $I < N$, buyers must purchase at a higher rate in these states.

An immediate implication of Corollary 1 is that by increasing the amount of historical information available to buyers (by increasing N), the equilibrium outcome can come arbitrarily close to first-best. Note that this result does not carry over to an environment with a fixed price. With a fixed price, the seller will play a mixed strategy ($\sigma_S = p_F$) in states $I < N$, also in the limit as $N \to \infty$.

3.4 Efficient equilibria when buyers post price

When buyers post price, and in an efficient equilibrium, the seller always provides high quality and buyers always purchase on the equilibrium path. The off-path price proposals are such that the seller is indifferent between high and low quality. Consequently, the threat of a lower

[26]Note that this could not be archived simply by setting the price to 1 exogenously, because this would imply that buyers would not purchase in state $I = N$, which in turn would result in the unraveling of reputational concerns.

price following a choice of low quality induces the seller to provide high quality. Furthermore, because the seller is indifferent, she can condition her quality choice on the proposed price. Because the threat of low quality following a low price proposal is credible, the seller can earn strictly positive payoffs in equilibrium. Proposition 2 makes the formal statement.

Proposition 2. ***Buyer-posted prices:*** *Assume that* $\mu \in [0, \frac{\delta - c}{\delta}]$ *and* $\delta > c$. *Then, for any* $N \geq 1$, *any efficient equilibrium is fully characterized by:*

> ***Price proposals:*** $p(I) = p' \in [\frac{c}{\delta}, 1 - \mu] \ \forall \ I \in \{1, ..., N\}$ *and* $p(0) = \frac{\delta p' - c}{\delta}$.
>
> ***Quality choice:*** $\sigma_S(I) = p(I)$ *for all* $I \in \{0, ..., N-1\}$; $\sigma_S(N) = 1$ *if* $p(N) \in [p', 1]$, $\sigma_S(N) = \frac{p(N) - \mu}{1 - \mu}$ *if* $p(N) \in [\mu, p')$, *and* $\sigma_S(N) = 0$ *otherwise.*
>
> ***Beliefs:*** $\mu(I) = 0 \ \forall \ I \in \{0, 1, ..., N-1\}$ *and* $\mu(N) = \mu$.
>
> ***Purchasing decision:*** $\sigma_B(I) = 1$ *for all* $I \in \{0, ..., N\}$ *and* $p(N) \in [0, 1]$.

The proof of Proposition 2 is given in appendix A.5. As Proposition 2 demonstrates, efficient equilibria have very different characteristics when buyers post price, as compared to when the price is exogenously fixed or when the seller posts price.[27] Below I consider some aspects of the result.

First, notice that there is a range of price proposals in state N that are consistent with an efficient equilibrium. The lower bound of this range, $\frac{c}{\delta}$, is such that the price posted in state $I = 0$ is zero. The upper bound of the range, $1 - \mu$, is such that buyers are exactly indifferent between posting this price and $p(N) = 0$.[28] Because the lower bound of the price range is bounded above c, the seller earns strictly positive payoffs in any efficient equilibrium when buyers post prices. Furthermore, the seller may earn higher payoffs when buyers post price than when the seller posts price. When the seller posts price, the per-period payoff is bounded above by $\mu(N) = \frac{\mu(N+1)}{\mu N + 1}$. When buyers post price, the per-period payoff is $p' - c$, where $p' \in [\frac{c}{\delta}, 1 - \mu]$. Thus, there are parameters such that for some $p' \in [\frac{c}{\delta}, 1 - \mu]$, $p' - c > \frac{\mu(N+1)}{\mu N + 1}$.

Second, with buyer-posted prices the seller provides high quality with certainty in all states. This contrasts with the equilibrium in Proposition 1 where the seller provides low quality with certainty in state $I = N$. When the seller proposes price, or when the price is

[27] The seller does not have the option to simply reject the offer. This is a simplification which is without consequence. To see this, assume that the seller does have the option to reject, in which case she earns a payoff of 0 and the index I resets to 0. Then clearly, accepting and providing low quality dominates rejecting the offer for all price proposals $p(I) \geq 0$.

[28] Note that for a large prior, i.e., if $\mu > \frac{\delta - c}{\delta}$, buyers will always prefer to offer a price of zero and so the equilibrium in Proposition 2 does not exist.

exogenously fixed, buyers can influence the seller payoffs through their purchasing decisions only. This limits the power of the dynamic incentives that buyers can provide in an equilibrium with seller-posted prices in two distinct ways. First, because the cost of quality is related to the purchasing decision, purchasing at a lower rate in state $I = 0$ implies that it is less costly for the seller to rebuild her reputation than it is to maintain it. Second, because the seller has an incentive to rebuild her reputation in states $I < N$, buyers have an incentive to purchase from a seller in these states. With buyer-posted prices, buyers can influence the benefit of building and maintaining a reputation by offering different prices depending on the state I while at the same time purchasing at the same rate in all states. Thus, buyers can provide dynamic incentives while at the same time keeping the cost of building a reputation at the same level as the cost of maintaining it.

4 Robustness and discussion

In this section, I discuss some of the key assumptions in the model and demonstrate that they are not, for the most part, critical to my main results.

4.1 The committed seller

Endogenizing prices in a model with a committed seller implies that I need to make assumptions regarding the way the committed seller sets the price and responds to prices.[29] As it turns out, I can arrive at my main results (Propositions 1 and 2) without assuming the presence of a committed seller. In section B of the online appendix, I assume instead that there are two strategic types which differ with respects to the cost of quality. In equilibrium under these alternative assumptions, the low-cost seller behaves as the committed seller type in the baseline model and the high cost type behaves as the strategic seller.

An alternative to my assumption regarding the pricing behavior of the committed seller would have been to assume that she was committed to a specific price, i.e., that the committed seller always posted a specific price and provided high quality if the price was equal to or above this price. Assuming that the committed seller was committed to a specific price would not dramatically alter the results, but would not be without consequence. With seller-posted prices, the key difference would be that the price posted in state $I = N$ was independent

[29]Note that by assumption, the committed seller maximizes revenue, which implies that she does not have costs, or has costs but disregards them. If I had assumed that the committed seller internalized cost, the equilibrium characterized in Proposition 1 would exist only for sufficiently long histories (N). Specifically, the equilibrium characterized in Proposition 1 would exist if $\frac{\mu(N+1)}{\mu N+1} \geq c$.

of the history length (N) and equal to the price the committed seller was committed to. With buyer-posted prices, it would result in the existence of an equilibrium with work-shirk dynamics when buyers post price, in addition to the equilibria characterized in Proposition 2.

4.2 Monitoring structure

I make several assumptions regarding the monitoring structure in the baseline model. First, buyers observe the previous quality choices of the seller regardless of decisions by previous buyers. Second, buyers do not observe the price history. Third, buyers do not observe the actions of previous buyers.

Consider first the assumption that buyers observe the previous quality choices of the seller regardless of decisions by previous buyers. In the following I argue that this assumption can be interpreted as a simplification of a slightly richer model in which information is more likely to be generated if purchases take place and where buyers make a decision on quantity in an interval. First, assume that rather than buyers making a binary choice, buyers decide on a quantity to purchase between zero and one. I let $\sigma_B(I)$ denote this quantity.[30] This implies that the seller earns a payoff of $\sigma_B(I)(p(I)-c)$ when a buyer purchases a quantity of $\sigma_B(I)$. Second, assume that information regarding quality in a particular period is generated with certainty if $\sigma_B(I) > 0$ in that period and with probability $1 > \epsilon > 0$ if $\sigma_B(I) = 0$.[31] The equilibria characterized in Propositions 1 and 2 will survive as equilibria of this alternative model, even if $\varepsilon = 0$, because $\sigma_B(I) > 0$ in all states in these equilibria. Furthermore, since information is generated with a positive probability regardless of $\sigma_B(I)$, and since the cost of quality is zero if $\sigma_B(I) = 0$, this alternative model cannot support equilibria in which the seller always provides high quality when the seller posts price. Consequently, the assumption that buyers observe the previous quality choices of the seller regardless of decisions by previous buyers does not in itself restrict the applications of the model.

Next, consider the assumptions that buyers do not observe the price history. Having buyers observe N previous prices would have implications for the equilibrium characterized in Proposition 1. Specifically, the price profile $\boldsymbol{p} = \{1, 1, 1, ..., \mu(N)\}$ could no longer constitute an equilibrium price profile, because this price profile reveals that the seller is strategic. In an equilibrium, the seller would therefore have to post the same price in all states. Because the seller plays according to $\sigma_S(I) = p(I)$ in states $I < N$, this price would have to be such

[30]Alternatively, $\sigma_B(I)$ could be interpreted as the share of a unit measure of consumers who purcahse.
[31]Levine (2021) makes a similar assumption.

that it was equal to the belief about the committed seller in state $I = N$.[32] Such a price will always exist and converges to 1 as $N \to \infty$. As such, Corollary 1 would still hold.

Last, I assume that buyers do not observe the actions of previous buyers. This assumption is reasonable for many applications of the model, e.g., ratings on platforms such Yelp and Tripadvisor will typically relate to actions of sellers and not buyers. The assumption, however, is not without consequence. In particular, letting buyers observe the actions of previous buyers will limit the seller´s ability to rebuild her reputation.[33]

4.3 Multiple price proposals

In the baseline model, I assume that either the seller or buyers have full influence over price. Thus, intermediate cases where both sides of the market have some influence are not covered. To check for robustness, I consider an alternative model where I assume that both the seller and the buyers post a price and each proposal is then observed by both players and implemented with a certain probability.[34] My results are robust in the sense that they do not require that either the seller or the buyers have full influence over price. I show that the equilibrium characterized under the assumption that the seller posts price exists when the probability that the price of the seller will be implemented is below 1, and that the equilibria characterized under the assumption that buyers post price exist when the probability that the price of the seller will be implemented is above 0.

5 Conclusion

I endogenize the price in a dynamic experience goods model where short-lived buyers choose between buying a good or not, and where a long-lived seller chooses between high or low quality. The price is endogenized by endowing either the seller or the buyer with the ability to post price. I show that outcomes depend crucially on which side of the market posts price.

With seller-posted prices, equilibrium behavior resembles that of a model with a fixed exogenous price: Equilibrium behavior follows a cyclical pattern where the strategic seller builds reputation for a finite number of periods only to exploit it. Unlike the model with

[32]Specifically, this price is pinned down by $\frac{\mu(1-p^{N+1})}{\mu(1-p^{N+1})+(1-\mu)p^N(1-p)} = p$, where the LHS is the belief about the committed seller in state $I = N$ when $\sigma_S(I) = p$ in states $I < N$.

[33]Liu (2011) demonstrates that information on the play of the short-lived player will lead to unraveling of cyclical equilibria. Pei (2022a) also emphasizes the role of information on the play of the short-lived player in models with limited memory.

[34]The analysis can be found in section C of the online appendix.

a fixed exogenous price, however, equilibrium outcomes with seller-posted prices can come arbitrarily close to first-best simply by increasing the observable history length.

When buyers post price, any efficient equilibrium has the features that the seller always provides high quality and buyers always purchase on the equilibrium path. Furthermore, the observable history length plays a smaller role because buyers need to observe only the last quality choice for equilibria to be efficient.

My results have implications for how trade in markets that rely on reputational concerns should be organized. In particular, organizing trade in a way such that buyers have sufficient influence over price may ensure efficiency while at the same time giving sellers at least a part of the surplus.

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A Proofs

A.1 Proof of Lemma 1

I start by verifying the following claims in order to help with the proof to follow: If equation (7) holds with a strict inequality and the seller posts price, $p(I) = 1$ is a best response in any state $I \leq N$. If equation (7) holds with a strict inequality and buyers post price, $p(I) = 0$ is a best response in any state $I \leq N$. Furthermore, if equation (7) holds with a strict inequality in any state I, $\sigma_B(I) = 1$ is a best response independent of the location of bargaining power. First, consider the case where the seller posts price. Equation (7) holds independent of $p(I)$, implying that buyers prefer to purchase for any $p(I) \leq 1$. Thus, the seller maximizes payoffs by proposing $p(I) = 1$. Next, consider the case where buyers post price. Again, equation (7) holds independent of $p(I)$, which implies that a strategic seller prefers high quality for any $p(I)$. Finally, if the seller strictly prefers high quality, the best response of the buyer must be $\sigma_B(I) = 1$.

Because $p(I) = 0 \; \forall I \in \{0, 1, ..., N\}$ when buyers post price, if equation (7) holds with a strict inequality, there cannot be an equilibrium where equation (7) holds with a strict inequality when buyers post price.

The rest of the proof is devoted to the case where the seller posts price. First, I show that high quality cannot be a strict best response in *all* states. The argument is straightforward: If high quality is a strict best response in all states, purchasing must be a best response of buyers in all states. Furthermore, any deviation from high quality would have to be followed by reversion to high quality, implying that the continuation payoffs are identical across all states, i.e., $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) \forall I \in \mathbf{I}$, and because $c > 0$, the condition in equation (7) cannot hold.[35] Note that this argument holds for any history length.

Finally, I show that high quality cannot be a strict best response in any state(s). I do so by constructing several contradictions based on three separate cases. These cases, and their respective implications for play in other states, will show up in any equilibrium where high quality is a strict best response in any number of states between 1 and $N - 1$.

Case 1: First, I show that there cannot be an equilibrium in which high quality is a strict best response in some state I if low quality is a best response in states $I - 1$ and I^+. Under the assumption of these best responses, the following must hold:

a) $\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \leq c$,

b) $\delta(V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$, and

c) $\delta(V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \leq c$,

where $I^+ = \min\{I + 1, N\}$ and $I^{++} = \min\{I^+ + 1, N\}$. Note first that if $N < 3$, there is an immediate contradiction because $N < 3$ must imply that $I = I^+$ or $I^+ = I^{++}$. For $N \geq 3$ I need to take further steps to arrive at a contradiction. First, note that b) implies that $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p(I) - c + \delta V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})$, and that a) implies that $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I^+)p(I^+) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$. Furthermore, b) and c) imply that $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$. By replacing $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})$ and $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ in $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$, I obtain the

[35] Note that this could change if other commitment types were included in the model. For example, if I included a type that was committed to low quality, buyers could believe with probability 1 that the seller was such a type after observing $I = 0$. This implies that high quality could be a strict best response in every state without $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) \; \forall \; I \in \mathbf{I}$.

following condition:

$$\sigma_B(I^+)p(I^+) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) > p(I) - c + \delta V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})$$
$$\updownarrow$$
$$c + \sigma_B(I^+)p(I^+) - p(I) > \delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})).$$

Because $\sigma_B(I^+)p(I^+) - p(I) \leq 0$ regardless of I^+ and I, $c + \sigma_B(I^+)p(I^+) - p(I) > \delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}))$ directly contradicts b).

As such, a), b), and c) cannot all be satisfied in an equilibrium, implying that there is no equilibrium in which high quality is a strict best response in some state I, while low quality is a best response in states $I-1$ and I^+.

Case 2: Next, I show that there cannot be an equilibrium in which high quality is a strict best response in some states I and I^+, and low quality is a weak best response in state $I-1$. Under the assumption of these best responses, the following must hold:

a) $\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$,

b) $\delta(V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$, and

c) $\delta(V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \leq c$.

First, note that a) implies that $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p(I^+) - c + \delta V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p})$. Then the condition $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$, which must still hold under b) and c), can be rewritten as

$$p(I^+) - c + \delta V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) > p(I) - c + \delta V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})$$
$$\updownarrow$$
$$\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})) > p(I^+) - p(I).$$

If the seller posts price, $p(I) = p(I^+) = 1$. Consequently, $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ if $\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})) > 0$. This can be true only if high quality is a strict best response in state I^{++}. If that is not the case, it would imply that

$$\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p})) > 0$$
$$\updownarrow$$
$$\sigma_B(I^{++})p(I^{++}) - 1 + c > \delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}))$$

which directly contradicts a) because $\sigma_B(I^{++})p(I^{++}) - 1 + c \leq c$. Thus, for a), b), and c) to hold when the seller posts price, high quality must be a best response in I^{++}.

By substituting for $V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p})$ and $V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p})$ in $\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p})) > 0$, I can show that high quality can be a best response in state I^{++} only if it is also a best response in $\min\{I^{++} + 1, N\}$. Following this argument, I find that for a), b), and c) to hold when the seller posts price, $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p})$ must be ever-increasing. However, because the number of states is finite, incentives cannot be ever-increasing.

The above implies that there is no equilibrium where high quality is a strict best response in some state I and I^+, and where low quality is a weak best response in state $I - 1$. Note that this result implies there cannot be an equilibrium where low quality is a weak best response in states 0 to n and high quality is a strict best response in the remaining $N - n$ states. The argument covers all history lengths $N \geq 1$. The key point is that if high quality is a strict best response in some states I, and low quality is a weak best response in state $I - 1$, incentives must be ever-increasing, which is impossible with a finite number of states.

Case 3: Last, I show that there cannot be an equilibrium in which high quality is a strict best response in some states I and $I - 1$, and low quality is a weak best response in state I^+. Under the assumption of these best responses, the following must hold:

a) $\delta(V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) \leq c$,

b) $\delta(V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$, and

c) $\delta(V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$.

Under a), b), and c), high quality must be a strict best response in every state before I. If not, I will again run into case 2. If high quality is a strict best response in all states before I, it must be the case that high quality is a strict best response in state 0, i.e., that $\delta(V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) > c$, which implies that $V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ must hold. If the seller posts price, I can verify $V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ holds if $V_S^*(2, \boldsymbol{\sigma}_B, \boldsymbol{p}) >$

$V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p})$:

$$V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) > 0$$
$$\downarrow$$
$$1 - c + \delta V_S^*(2, \boldsymbol{\sigma}_B, \boldsymbol{p}) - (1 - c) + \delta V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > 0$$
$$\downarrow$$
$$V_S^*(2, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > 0.$$

Thus, high quality can be a strict best response in state 0 if $V_S^*(2, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > 0$. Continuing this process, under the assumption that high quality is a strict best response in every state before I, I find that $V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})$ holds if $V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p})$. However, $V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p})$ is a direct contradiction of $V_S^*(I^{+}, \boldsymbol{\sigma}_B, \boldsymbol{p}) > V_S^*(I^{++}, \boldsymbol{\sigma}_B, \boldsymbol{p})$, which is an immediate implication of a) and b). As such, when the seller posts price, statements a), b), and c) cannot hold simultaneously.

Note again that this argument covers all history lengths $N \geq 1$. The key point is that if high quality is a strict best response in I, it must also be so in I^+.

As stated earlier, the sequences covered in cases 1–3, and their implications for play in other states, will show up in any equilibrium where high quality is a strict best response in any number of states between 1 and $N-1$. That is, if high quality is a strict best response in some state I, but low quality is a best response in state $I-1$, I run into case 1 if low quality is a weak best response in state I^+ and run into case 2 if high quality is a strict best response in state I^+. Furthermore, if high quality is a strict best response in some state $I-1$, but low quality is a weak best response in state I, I run into case 1 if low quality is a weak best response in state $I-2$ and run into case 3 if high quality is a strict best response in state $I-2$, independent of the seller's best response in state I^+. If high quality is a strict best response in states $I-1$, I, and I^+, I will unavoidably run into case 2 or case 3 unless high quality is a strict best response in all states, and I have already shown that this cannot be an equilibrium. Consequently, there cannot be an equilibrium where high quality is a strict best response.

A.2 Proof of Lemma 2

If the seller is indifferent in state $I \in \mathbf{I}$, then

$$V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p(I)\sigma_B(I) + V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p(I)\sigma_B(I) - c\sigma_B(I) + V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}).$$

Likewise, if the seller has a strict preference for low quality in state I, then

$$V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p(I)\sigma_B(I) + V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) > p(I)\sigma_B(I) - c\sigma_B(I) + V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}).$$

In either case, $V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I)p(I) + V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})\ \forall\, I \in \mathbf{I}$. Consequently, $V_S^*(I^+, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) = \sigma_B(I^+)p(I^+) - \sigma_B(0)p(0)$ follows directly.

A.3 Proof of Proposition 1

The proof consists of two parts. In part 1 I verify that the strategy profile in Proposition 1 constitutes a stationary PBE. In part 2 I verify that any efficient equilibrium must have the characteristics listed in Proposition 1.

1. Consider first the committed seller. By posting a price $p(N) < \frac{\mu(N+1)}{\mu N+1}$ in state N, the committed seller would earn strictly lower payoffs: The buyer either purchases with certainty if $p(N) \leq \phi(p(N), N)$ or mixes according to $\sigma_B(N-1)$ if $p(N) > \phi(p(N), N)$. Consequently, posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ results in higher payoffs than posting any lower price. By posting a price, $p(N) > \frac{\mu(N+1)}{\mu N+1}$ in state N, the buyer will play as if in state $N-1$. Posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ strictly dominates any price $p(N) > \frac{\mu(N+1)}{\mu N+1}$ if $\frac{\mu(N+1)}{\mu N+1} > \sigma_B(N-1)$.[36] I show that the inequality must hold later in the proof.

   The rest of the proof proceeds as follows: I first verify that the strategies and beliefs in state $I = N$ can be a part of an equilibrium. I then consider states $I < N$.

   Strategies and beliefs in state $\boldsymbol{I = N}$: Conditional on $\sigma_S(I) = p(I) = 1\ \forall\, I < N$, the probability of a strategic seller being in state N is $\lambda(N) = \frac{1}{1+N}$. Using $\lambda(N) = \frac{1}{1+N}$ together with equation 2, I find that $\mu(N) = \frac{\mu(N+1)}{\mu N+1}$. Conditional on $\sigma_S(N) = 0$, $\sigma_B(N) = 1$ and $\phi(I, p)$, I now verify that the best response of the seller is to set the price at the buyers' reservation price, i.e., $p(N) = \frac{\mu(N+1)}{\mu N+1}$, which is the same price as the committed seller posts in this state. First, by posting a price $p(N) < \frac{\mu(N+1)}{\mu N+1}$ in state N the seller earns strictly lower payoffs. Second, by posting a price, $p(N) > \frac{\mu(N+1)}{\mu N+1}$ in

[36] Because setting the price equal to 1 must be the optimal deviation.

state N, the buyer will play as if in state $N-1$ (these strategies are derived below) and the seller plays a mix such that the buyer is indifferent. Posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ strictly dominates any price $p(N) > \frac{\mu(N+1)}{\mu N+1}$ if

$$\mu(N) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p} \geq \sigma_B(N-1) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}),$$

which holds if $\frac{\mu(N+1)}{\mu N+1} \geq \sigma_B(N-1)$. I show that the inequality must hold later in the proof.

Next, because buyers are indifferent if $p(N) = \frac{\mu(N+1)}{\mu N+1}$, $\sigma_B(N) = 1$ is a best response. Furthermore, given the off-path beliefs and the strategy of the seller following an off-path price proposal, the strategy of buyers constitutes a best response. For $\sigma_S(N) = 0$ to be a best response, the following condition must be satisfied:

$$\delta(V_S^*(N, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) = \delta(p(N) - \sigma_B(0)) \leq c.$$

The first equality in the condition above uses Lemma 2. Rewriting this condition, I find that low quality is a best response in $I = N$ if $\sigma_B(0) \geq \frac{\delta p(N)-c}{\delta}$. Using equation 9 to substitute out $\sigma_B(0)$:

$$\frac{p(N)}{\frac{\delta}{\delta-c}\left(1-\left(\frac{c}{\delta}\right)^{N+1}\right)} \geq \frac{\delta p(N)-c}{\delta}$$
$$\Updownarrow$$
$$p(N) \geq \frac{\delta p(N)-c}{\delta-c}\left(1-\left(\frac{c}{\delta}\right)^{N+1}\right).$$

Because $\left(1-\left(\frac{c}{\delta}\right)^{N+1}\right) < 1$ and $p(N) \geq \frac{\delta p(N)-c}{\delta-c}$ the condition holds. Thus, $\sigma_S(N) = 0$ is a best response.

Strategies and beliefs in states $\boldsymbol{I < N}$: I start by verifying that $p(I) = 1$ in all states $I < N$ constitutes a best response. I start by fixing some commonly known price sequence $\boldsymbol{p}'$ and a strategy $\boldsymbol{\sigma}'_B$ of buyers such that the seller is indifferent in all states $I < N$ and let the seller play according to $\sigma_S(I) = p(I)\ \forall\ I \in \{0, 1, ..., N-1\}$ and $\sigma_S(N) = 0$. Then, conditional on the price sequence $\boldsymbol{p}'$, $\boldsymbol{\sigma}_S$ and $\boldsymbol{\sigma}'_B$ are mutual best responses. Because $\boldsymbol{p}'$ is common knowledge, $\boldsymbol{\sigma}'_B$ is a function of $\boldsymbol{p}'$. It is, however, not a function of the price posted in any one period. That is, if the seller posts any other price, there is no reason for buyers to deviate: Because the seller's continuation payoffs are unchanged, the mix that makes the seller indifferent in that particular period must be unchanged. Thus, the seller posts the price that maximizes payoffs conditional on $\boldsymbol{\sigma}'_B$ and $\boldsymbol{p}'$, and because $\boldsymbol{\sigma}'_B$ is independent of this price, payoffs are maximized by setting the price to 1. Now, because $\boldsymbol{p}'$ in equilibrium must correspond to optimal price proposals, $\boldsymbol{p}' = \{1, 1, 1, ..., 1\}$.

Using $p(I) = 1$ for all states $I < N$, I derive the equilibrium strategy of the buyers. First, note that for $\sigma_S(I) = p(I)$, buyers are indifferent in states $I < N$. As such, any mix is a best response. To determine $\sigma_B(I)$ for $I < N$ that also makes the seller indifferent, I use equation (8) and consider any integer $I \in [0, N)$. Then, if the seller is indifferent between high and low quality in state I, the following condition must hold:

$$\delta(\sigma_B(I+1) - \sigma_B(0)) = c\sigma_B(I)$$
$$\Updownarrow$$
$$\sigma_B(I) = \frac{\delta(\sigma_B(I+1) - \sigma_B(0))}{c}.$$

This implies that $\sigma_S(I) = p(I)$ is a best response if there is a $\sigma_B(I)$ that makes the seller indifferent in states $I < N$. At $I = 0$, the condition is slightly different because this is the only state, apart from N, that the seller can remain in from one period to the next. In state $I = 0$, the condition in equation (8) becomes

$$\delta(\sigma_B(1) - \sigma_B(0)) = c\sigma_B(0)$$
$$\Updownarrow$$
$$\sigma_B(0) = \frac{\delta\sigma_B(1)}{1+c}.$$

To determine $\sigma_B(0)$ as well as the sequence $\{\sigma_B(n)\}_{I=1}^{N-1}$, I start at $N-1$, taking $\sigma_B(0)$ as given, and then applying backward induction. This process results in equation (9).

Next, I verify that the sequence $\{\sigma_B(I)\}_{I=0}^{N-1}$ is well defined (bounded within the unit interval). I do so in two steps. First, I show that $\sigma_S(0)$ is bounded within the unit interval. Second, I show that $\{\sigma_B(I)p(I)\}_{I=0}^{N-1}$ is an increasing sequence that is bounded below by $\sigma_B(0)$ and above by $p(N)$.

First step: Because $p(N) \in (0, 1)$, $\sigma_S(0) \in (0, 1)$ if $k(N+1) = \frac{\delta}{\delta - c}\left(1 - \left(\frac{c}{\delta}\right)^{N+1}\right) \geq 1$. Because $\delta > c$, this inequality will hold for all $N \geq 1$.

Second step: Next I show that $\{\sigma_B(I)\}_{I=0}^{N-1}$ is an increasing sequence that is bounded below by $\sigma_B(0)$ and above by $p(N)$. I start at $N-1$, using equation 8. Note that $\sigma_B(N)p(N) = p(N)$ and $\sigma_B(N-1)p(N-1) = \sigma_B(N-1)$ in equilibrium. Consequently, $\sigma_B(N)p(N) = p(N) > \sigma_B(N-1)$ if

$$\sigma_B(N-1) = \frac{\delta(p(N) - \sigma_B(0))}{c} < p(N)$$
$$\downarrow$$
$$p(N)(\delta - c) < \delta\sigma_B(0).$$

By replacing $\sigma_B(0)$ I can rewrite this to

$$p(N)(\delta - c) < \frac{p(N)(\delta - c)}{\left(1 - \left(\frac{c}{\delta}\right)^{N+1}\right)}$$
$$\downarrow$$
$$1 < \frac{1}{\left(1 - \left(\frac{c}{\delta}\right)^{N+1}\right)}.$$

Because $\left(\frac{c}{\delta}\right)^N \in (0, 1)\ \forall\ N \geq 1$, this inequality must hold. As such, $\sigma_B(N-1) < p(N)$, where $p(N) = \frac{\mu(N+1)}{\mu N+1}$. Next, because $\sigma_B(N-1) < p(N)$, it must be the case that

$$\sigma_B(N-2) = \frac{\delta(\sigma_B(N-1) - \sigma_B(0))}{c} < \sigma_B(N-1).$$

By induction, this holds for for all states. Consequently, $\{\sigma_B(I)\}_{I=0}^{N-1}$ is increasing and bounded by $p(N)$. I have already shown that $\sigma_B(0)$ is positive for all $N \geq 1$, so this completes the proof.

2. In the following, I verify that the stationary PBE in Proposition 1 constitutes a unique efficient equilibrium. First, note that the model does not support equilibria in which the seller has strict preferences for high quality (Lemma 1). Thus, the only remaining equilibrium candidates where the seller provides high quality on the equilibrium path are those in which the seller is indifferent.

   The rest of the proof proceeds in four steps. In the first step, I show that buyers cannot condition their strategy in states $I < N$ on the current price proposal. This implies that $p(I) = 1$ in states $I < N$ in any equilibrium where buyers purchase with a positive probability in states $I < N$ and that $\sigma_S(I) = 1$ in states $I < N$. This again implies that $\mu(N) = \frac{\mu(N+1)}{\mu N+1}$ are the only beliefs that can be consistent with equilibrium play when buyers purchase in states $I < N$, that states $I < N$ are on the equilibrium path, and that the seller provides low quality with certainty in state $I = N$. In the second step, I verify that there cannot be an equilibrium where the seller provides high quality in state $I = N$. In the third step, I show that there cannot be an equilibrium where either or both seller types mix between prices in state $I = N$ in a way such that $p(N)\sigma_B(N) \geq \mu(N)$ and $\phi(p(N), N) > \mu(N)$. In step four, I show that $\sigma_B(I)$, in the stationary PBE in Proposition 1 and in states $I < N$, is increasing in $p(N)$.

Step 1: I start by assuming some strategy profile ($\boldsymbol{\sigma}_S$, $\boldsymbol{\sigma}_B$, $\boldsymbol{p}$) and some sequence of beliefs $\boldsymbol{\mu}$ induced by said strategy profile. If the strategy profile constitutes an equilibrium, it ensures that buyers and the seller, given $\boldsymbol{p}$, are indifferent in each state $I < N$. Now, assume that $\boldsymbol{\sigma}_B$ is conditioned on the current price in the following way: If a price proposal corresponds to the one suggested by $\boldsymbol{p}$, buyers mix such that the seller is indifferent between high and low quality, otherwise the buyer plays some other mix. Now consider what happens if the seller deviates and proposes some price not corresponding to $\boldsymbol{p}$. As prescribed by $\boldsymbol{\sigma}_B$, the buyers then play some mix such that the seller is no longer indifferent between high and low quality: The seller has a strict best response of either high or low quality. However, if the seller provides either high or low quality with certainty, the mix prescribed by $\boldsymbol{\sigma}_B$ in case of deviations from $\boldsymbol{p}$ cannot be a best response. As such, the strategy profile ($\boldsymbol{\sigma}_S$, $\boldsymbol{\sigma}_B$, $\boldsymbol{p}$) cannot constitute equilibria. Consequently, there cannot be equilibria in which buyers condition their strategy on the current price proposal in states $I < N$.

Step 2: For the seller to provide high quality in state $I = N$, the seller would have to be indifferent in that state. In the following, I show that there cannot be an equilibrium where the seller is indifferent in state $I = N$. For the seller to be indifferent in state $I = N$, the following condition has to hold:

$$\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) = c\sigma_B(N)$$

. In state $I = N - 1$, the seller is indifferent if

$$\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) = c\sigma_B(N - 1).$$

Note that the seller has to be indifferent in state $I = N - 1$ if she is indifferent in state

$I = N$. If she preferred low quality, buyers would not purchase in state $I = N - 1$, which in turn would imply that $\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) < 0$. If $\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) < 0$, the seller cannot be indifferent in state $I = N$ because $\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) < 0$ would imply that $\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) < c\sigma_B(N)$. Furthermore, because the RHS in the two conditions above are identical, they can hold simultaneously only if $\sigma_B(N-1) = \sigma_B(N)$. Using $\sigma_B(N-1) = \sigma_B(N)$, along with the property that the seller will post a price of 1 in state $I = N-1$, the seller is indifferent in state $N-2$ if

$$\delta(\sigma_B(N) - p(0)\sigma_B(0)) = c\sigma_B(N-2).$$

Note again that the seller has to be indifferent in state $I = N-2$ if she is indifferent in state $I = N$. If she preferred low quality, it would imply that $\delta(\sigma_B(N) - p(0)\sigma_B(0)) < 0$, which in turn would imply that $\delta(p(N)\sigma_B(N) - p(0)\sigma_B(0)) < c\sigma_B(N)$. Note also that $\sigma_B(N-2)$ must be larger or equal to $\sigma_B(N)$ (equal if $p(N) = 1$ and larger if $p(N) < 1$). Regardless, continuing this process backwards ultimately results in $p(N)\sigma_B(N) < p(0)\sigma_B(0)$. If $p(N)\sigma_B(N) < p(0)\sigma_B(0)$, there is no $\sigma_B(N)$ that can make the seller indifferent in state $I = N$. As such, there cannot be an equilibrium where the seller is indifferent in state $I = N$.

Step 3: I consider an equilibrium candidate in which the committed seller plays a mixed strategy over a set of prices (possibly contentious) and where the payoff from the mixed strategy is at least $\mu(N)$, and then show that this leads to a contradiction. I let P denote the set of price proposals over which the committed seller mixes. Because the committed seller is mixing, she must be indifferent between all the prices in P. Furthermore, a mixed strategy of the committed seller must imply that buyers are also mixing and that they are indifferent to all price proposals in P. Indifference on the part of buyers implies that $p(N) = \phi(p(N), N)$ for all price proposals in P. By assumption, the payoff from this mixed strategy must be at least $\mu(N)$, and so the lowest price in P would have to be at least $\mu(N)$. In equilibrium, the strategic seller would never post a price below the lowest price in P because this would earn her strictly lower payoffs, as compared to, e.g., posting a price at the lowest price P, regardless of off-path beliefs. Consequently, the strategic seller would best respond by playing a mix over the same set as the committed seller. However, there is no mix of the strategic seller such that the buyer will be indifferent to all the prices in the set: If the strategic seller mixes with the same probabilities as the committed seller, then $\phi(N, p) = \mu(N)$ for all prices in the set, which implies that not purchasing is a strictly best response of buyers for all $p(N) > \mu(N)$. If the strategic seller mixes with different probabilities, then it must be the case that $\phi(N, p) < \mu(N)$ for at least some price proposals in P. For such price proposals, the best response of buyers is to not purchase, which contradicts buyers being indifferent to price proposals in P.

Step 4: From equation 9 it is clear that $\sigma_B(0)$, in equilibrium, is increasing in $p(N)$. Next, consider the condition in equation 8 in state $I = 0$:

$$\delta(\sigma_B(1) - \sigma_B(0)) = c\sigma_B(0).$$

Because $\sigma_B(0)$ is increasing in $p(N)$, $\sigma_B(1)$ must also be increasing in $p(N)$. Next, consider the condition in equation 8 in state $I = 2$:

$$\delta(\sigma_B(2) - \sigma_B(0)) = c\sigma_B(1).$$

Because $\sigma_B(0)$ and $\sigma_B(1)$ are increasing in $p(N)$, $\sigma_B(2)$ must also be increasing in $p(N)$. Continuing this argument forward leads to the result that $\sigma_B(I)$, in the stationary PBE in Proposition 1 and in all states $I < N$, is increasing in $p(N)$.

Finally, by step 1, buyers cannot condition their strategy on the price proposal in any current state, which implies that if buyers purchase with positive probability in an equilibrium, the seller will post a price of 1 in all states $I < N$ in that equilibrium. By step 2, the seller cannot, in equilibrium, provide high quality in state $I = N$. Together, steps 1 and 2 imply that an equilibrium where buyers purchase with positive probability has the characteristic that the seller provides high quality with certainty in all state $I < N$ and low quality with certainty in state $I = N$. Steps 3 and 4 together imply that out of all equilibria in which buyers purchase with positive probability, and the seller provides high quality with certainty in all state $I < N$ and low quality with certainty in state $I = N$, the equilibrium characterized in Proposition 1 is the only one that is constrained efficient.

A.4 Proof of Corollary 1

In the limit, as $N \to \infty$, $\mu(N) \to 1$ and so the equilibrium price proposal in state $I = N$ converges to 1. Next, $\lim_{N\to\infty} \sigma_B(0) = \lim_{N\to\infty} \frac{p(N)}{k(N+1)} = \frac{\delta - c}{\delta}$. Using these limits, I can show that $\sigma_B(N - i)$ converges to 1 as $N \longrightarrow \infty$ for any positive integer $i < N$.

From the equation (8),

$$\delta(\sigma_B(N-i+1)-\sigma_B(0)) = c\sigma_B(N-i)$$

must hold in all states $I < N$. Taking the limit as $N \longrightarrow \infty$ while keeping i constant results in

$$\delta(\lim_{N\longrightarrow\infty}\sigma_B(N-i+1)-\frac{\delta-c}{\delta}) = c\lim_{N\longrightarrow\infty}\sigma_B(N-i),$$

which will hold only for $\lim_{N\longrightarrow\infty}\sigma_B(N-i+1) = \lim_{N\longrightarrow\infty}\sigma_B(N-i) = 1$. Because the seller is indifferent in any state $I < N$ and prefers low quality in state N, the discounted payoff $V_S^*(N-i,\boldsymbol{\sigma}_B,\boldsymbol{p})$ for any positive integer $i \leq N$, in the limit, is given by

$$\lim_{N\longrightarrow\infty}V_S^*(N-i,\boldsymbol{\sigma}_B,\boldsymbol{p}) = 1+\delta\lim_{N\longrightarrow\infty}V_S^*(0,\boldsymbol{\sigma}_B,\boldsymbol{p}) = 1+\frac{\delta-c}{1-\delta}.$$

Normalizing this by multiplying by $1-\delta$ results in

$$(1-\delta)\lim_{N\longrightarrow\infty}V_S^*(N-i,\boldsymbol{\sigma}_B,\boldsymbol{p}) = 1-c,$$

which is equal to the per-period Stackelberg payoffs. In state $I = 0$, the seller's normalized payoffs are $\frac{\delta-c}{\delta}$, which converges to $1-c$ when $\delta \longrightarrow 1$. This completes the proof.

A.5 Proof of Proposition 2

The proof consists of two parts. In part 1 I verify that the strategy profile in Proposition 2 constitutes stationary PBEs. In part 2 I verify that the stationary PBE is a unique efficient equilibrium:

1. I first verify that the strategy of the seller constitutes a best response given the strategy of buyers. First, note that $p(0) = \frac{\delta p'-c}{\delta} \geq 0$ for $p' \geq \frac{c}{\delta}$. Furthermore, in any state $I \in \{0,...,N\}$, given the strategy of buyers, the condition in equation 8 becomes

   $$\delta(p'-\frac{\delta p'-c}{\delta}) = c$$

   which holds with equality for any p'. Because the seller is indifferent in all states, any strategy will constitute a best response.

   Next, I verify that the price proposals of buyers constitute a best response given the strategy of the seller. In state $I < N$, the seller plays a mix such that buyers are indifferent between purchasing and not purchasing, given any price proposal. Consequently, any price proposal constitutes a best response. In state $I = N$, the seller provides high quality with certainty if $p(N) \geq p'$. As such, $p(N) = p'$ yields higher payoffs to buyers than any $p(N) > p'$. Next, for any price $p(N) \in [\mu, p')$ the seller plays a mix such that buyers are indifferent between purchasing and not purchasing. Not purchasing results in a payoff of 0. Thus, $p(N) = p'$ results in a higher payoff than any $p(N) \in [\mu, p')$ if $1 - p' > 0$. This holds because p' is bounded above by $1 - \mu > 0$. Finally, for any $p(N) < \mu$, the seller provides low quality with certainty. This implies that the highest expected payoff that buyers can earn from a price proposal $p(N) < \mu$ is μ (by proposing $p(N) = 0$). Thus, $p(N) = p'$ results in a higher payoff than any $p(N) < \mu$ if $1 - p' \geq \mu$. This holds because p' is bounded above by $1 - \mu > 0$.

Finally, I verify that the purchasing decisions constitute a best response. In states $I < N$, given any price proposal $p(I) \in [0, 1]$ and $\sigma_S(I) = p(I)$, buyers are indifferent. As such, $\sigma_B(I) = 1$ is a best response. In state $I = N$, the strategy of the seller ensures that purchasing is a best response following any proposal.

2. I begin by verifying that no other combinations of price proposals can result in an equilibrium where the seller always provides high quality on the equilibrium path, and where buyers always purchase on and off the equilibrium path. First, note that the price proposals in Proposition 2 are such that the seller is exactly indifferent. Thus, any other combination of price proposals would lead to the seller either strictly preferring low quality or strictly preferring high quality. The former cannot be a part of an efficient equilibrium, because it is not constrained efficient, and the latter is ruled out by Lemma 1. Consequently, in an equilibrium where dynamic incentives come through price proposals, these price proposals must be as characterized in Proposition 2.

   Finally, I show that there cannot be an equilibrium where the seller always provides high quality on the equilibrium path and where $\sigma_B(I) < 1$ for some $I < N$. To do so, I use the indifference condition in equation 8 in states $I = N$ and $I = 0$:

$$\delta(p(N) - p(0)\sigma_B(0)) = c$$
$$\delta(p(1)\sigma_B(1) - p(0)\sigma_B(0)) = c\sigma_B(0).$$

   Here I have used the property that any equilibrium in which buyers do not purchase with certainty in state $I = N$ cannot be constrained efficient and so $\sigma_B(N) = 1$. Solving both of the above conditions for $\sigma_B(0)$ and equating the two resulting expressions yield:

$$c(\delta(p(N) - p(0)) - c) = \delta^2 p(0)(p(1)\sigma_B(1) - p(N)).$$

For the equality to hold, we need $p(1)\sigma_B(1) = p(N)$ and $\sigma_B(0) = 1$. To see this, consider the following: For $\sigma_B(0) < 0$ it must be the case that $p(1)\sigma_B(1) < p(N)$, in which case the RHS of the equality is strictly negative (from the condition for state $I = 0$). However, for $\sigma_B(0) < 0$ the LHS is strictly positive (from the condition for state $I = N$). Consequently, the equality hold only if $p(1)\sigma_B(1) = p(N)$ and $\sigma_B(0) = 1$. It follows that $\sigma_B(I) = 1$ in all other states $I \in \{2, ..., N-1\}$.

B Equilibrium play in $t < N$

In this appendix, I characterize equilibrium play in initial periods $t < N$.[37] Before moving on, I define a new index, $R(h,t)$, with a similar interpretation to that of I. I let $R(h,t)$ be defined as follows:

$$R(h,t) = \begin{cases} I(h) & \text{if } t \geq N \\ t & \text{if } \forall\ k \in \{1, 2, ..., t-1\}, h_k = 1, \text{ and } t < N \\ \min\{k : h_k = 0\} - 1 & \text{if } \min\{k : h_k = 0\} - 1 < t \text{ and } t < N. \end{cases}$$

As with $I(h)$, $R(h,t)$ counts the number of instances of high quality since the last instance of low quality. The difference is that $R(h,t)$ also takes into account that the history lengths in the initial periods are shorter than N.

I first consider equilibrium play in periods $t < N$ when the seller posts price. Consider the following candidate for initial equilibrium play: In all periods $t < N$, $p(R) = 1$ for both seller types, $\sigma_S(R) = \frac{p(R) - \phi(R,p(R))}{1-\phi(R,p(R))}$; $\mu(R) = \mu$ if $R = t$ and $\mu(R) = 0$ otherwise; $\phi(R, p(R)) = \mu(R)$ if $p(R) = 1$ and $\phi(R, p(R)) \leq \mu(R)$ otherwise; and buyers play according to the strategies described in Proposition 1, $\sigma_B(R) = \sigma_B(I)$ for $I = R$. Or in words, the seller plays a mix, such that buyers are indifferent given posterior beliefs, and offers a price equal to 1. Thus, the seller must provide high quality with probability 1. Because the seller plays like a committed player in $t < N$, it must be the case that $\mu(R) = \mu$ if $R = t$. Furthermore, buyers play a mix that ensures that the seller is indifferent: In Proposition 1 I established that the mix played

[37] Note that the time starts at 0, so at $t = N$ the buyers will observe N previous actions.

by buyers ensures that the seller is indifferent. The buyer arriving at $t = N$ infers only that $t \geq N$ and updates beliefs according to equation (2) and the seller posts a price such that the buyer is indifferent and provides low quality with probability 1. As such, the equilibrium candidate constitutes an equilibrium.

Next, I consider equilibrium play in period $t < N$ when buyers post price. Consider the following candidate for initial equilibrium play when buyers post price: In all periods $t < N$, $p(R) = p' \in [\frac{c}{\delta}, 1-\mu]$ if $R > 0$ or $R = t = 0$ and $p(R) = \frac{\delta p' - c}{\delta}$ otherwise; when $R > 0$ or $R = t = 0$, $\sigma_S(R) = 1$ if $p(R) \in [p', 1]$, $\sigma_S(R) = \frac{p(R)-\mu}{1-\mu}$ if $p(R) \in [\mu, p')$, and $\sigma_S(R) = 0$ otherwise, and when $R = 0$ or $t > 0$, $\sigma_S(R) = p(R)$; $\mu(R) = \mu$ if $R = t$ and $\mu(R) = 0$ otherwise; $\mu(R) = \mu$ if $R = t$ and $\mu(R) = 0$ otherwise; and $\sigma_B(R) = 1$ for all R. In the proof of Proposition 2, I verify that this strategy profile constitutes an equilibrium.

C Equilibrium with a fixed exogenous price

This appendix contains a characterization of equilibria under the assumption of a fixed exogenous price p_F. All the other details of the model are as described in section 2. Note that because Lemmas 1 and 2 do not assume any particular price profile, they will also hold for $\boldsymbol{p} = (p_F, p_F, ..., p_F)$.

Proposition 3. ***Fixed exogenous price:*** *Consider some exogenously fixed price profile $\boldsymbol{p} = (p_F, p_F, ..., p_F)$, and assume that $\mu > 0$ and $\delta p_F > c$. Then there is a $\hat{N}$ such that for any $N \geq \hat{N}$, there is a unique efficient equilibrium fully characterized by:*

Quality choice: $\sigma_S(N) = 0$ *and* $\sigma_S(I) = p_F \; \forall \; I \in \{0, 1, ..., N-1\}$.

Beliefs: $\mu(I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$ *and* $\mu(N) = \frac{\mu\left(1-p_F^{N+1}\right)}{\mu\left(1-p_F^{N+1}\right)+(1-\mu)p_F^N(1-p_F)}$.

Purchasing decision: $\sigma_B(N) = 1$ *if* $\mu(N) \geq p_F$ *and* $\sigma_B(N) = 0$ *otherwise, and* $\sigma_B(I) = \frac{\delta p_F(\sigma_B(I+1)-\sigma_B(0))}{c} \; \forall \; I \in \{1, 2, ..., N-1\}$, *and*

$$\sigma_B(0) = \frac{1}{r(N+1)} \tag{10}$$

and $r(x) = \frac{\delta p_F}{\delta p_F - c}\left(1 - \left(\frac{c}{\delta p_F}\right)^x\right)$.

Proof. The proof consists of two parts. In part 1 I verify that the strategy profile in Proposition 3 constitutes a stationary PBE. In part 2 I verify that any efficient equilibrium must have the characteristics listed in Proposition 3:

1. I first consider the strategies in state $I = N$. Conditional on $\sigma_S(I) = p_F \ \forall \ I < N$, the probability of a strategic seller being in state N is $\lambda(N) = \frac{p_F^N(1-p_F)}{1-p_F^{N+1}}$. Using $\lambda(N) = \frac{p_F^N(1-p_F)}{1-p_F^{N+1}}$ together with equation 2, I find that the belief about the committed type in state $I = N$ is $\frac{\mu\left(1-p_F^{N+1}\right)}{\mu\left(1-p_F^{N+1}\right)+(1-\mu)p_F^N(1-p_F)}$. Note that $\frac{\mu\left(1-p_F^{N+1}\right)}{\mu\left(1-p_F^{N+1}\right)+(1-\mu)p_F^N(1-p_F)}$ is increasing in N for any $\mu \in (0, c)$. As such, there is always a N that ensures that $\frac{\mu\left(1-p_F^{N+1}\right)}{\mu\left(1-p_F^{N+1}\right)+(1-\mu)p_F^N(1-p_F)} \geq p_F$ is satisfied.

   Next, because the buyers, conditional on beliefs, prefer to purchase, $\sigma_B(N) = 1$ is a best response. For $\sigma_S(N) = 0$ to be a best response, the following condition must be satisfied:

$$\delta(V_S^*(N, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) = \delta p_F(1 - \sigma_B(0)) \leq c.$$

   The strict equality in the condition above uses Lemma 2. Rewriting this condition, I find that low quality is a best response in $I = N$ if $\sigma_B(0) \geq \frac{\delta p_F - c}{\delta p_F}$. Using equation 10 to substitute out $\sigma_B(0)$ I obtain the following:

$$\frac{1}{\frac{\delta p_F}{\delta p_F - c}\left(1 - \left(\frac{c}{\delta p_F}\right)^{N+1}\right)} \geq \frac{\delta p_F - c}{\delta p_F}$$
$$\Updownarrow$$
$$\left(\frac{c}{\delta p_F}\right)^{N+1} \geq 0.$$

   Because $\delta p_F > c$ the inequality is strict for any $N > 0$. Thus, low quality is a strict best response.

   Next, I consider the strategies for $I < N$. I start by deriving the equilibrium strategy of the buyers. First, note that for $\sigma_S(I) = p_F$, buyers are indifferent in states $I < N$. As such, any mix is a best response. To determine $\sigma_B(I)$ for $I < N$ that also makes the seller indifferent, I use equation (8) (under the assumption of a fixed price) and consider any integer $I \in [0, N)$. Then, if the seller is indifferent between high and low quality in state I, the following condition must hold:

$$\delta p_F(\sigma_B(I+1) - \sigma_B(0)) = c\sigma_B(N)$$
$$\updownarrow$$
$$\sigma_B(I) = \frac{\delta p_F(\sigma_B(I+1) - \sigma_B(0))}{c}.$$

This implies that $\sigma_S(I) = p_F$ is a best response if there is a $\sigma_B(I)$ that makes the seller indifferent in states $I < N$. At $I = 0$, the condition is slightly different because this is the only state, apart from N, that the seller can remain in from one period to the next. In state $I = 0$, the condition in equation (8) becomes

$$\delta p_F(\sigma_B(1) - \sigma_B(0)) = c\sigma_B(0)$$
$$\updownarrow$$
$$\sigma_B(0) = \frac{\delta p_F \sigma_B(1)}{p_F + c}.$$

To determine $\sigma_B(0)$ as well as the sequence $\{\sigma_B(n)\}_{n=1}^{N-1}$, I start at $N-1$, taking $\sigma_B(0)$ as given, and then applying backward induction. This process results in equation (10). Next, I verify that the $\{\sigma_B(I)\}_{I=0}^{N-1}$ is well defined (bounded within the unit interval). I do so in two steps. First, I show that $\sigma_S(0)$ is bounded within the unit interval. Second, I show that $\{\sigma_B(I)\}_{I=0}^{N-1}$ is an increasing sequence that is bounded below by $\sigma_B(0)$ and above by 1.

First step: $\sigma_S(0) \in (0,1)$ if $r(N+1) = \frac{\delta p_F}{\delta p_F - c}\left(1 - \left(\frac{c}{\delta p_F}\right)^{N+1}\right) > 1$. Because $\delta p_F > c$, this inequality will hold for all $N \geq 1$.

Second step: Next I show that $\{\sigma_B(I)\}_{n=0}^{N-1}$ is an increasing sequence that is bounded below by $\sigma_B(0)$ and above by 1. I start at $N-1$, using $\sigma_B(N) = 1$. $1 > \sigma_B(N-1)$ if

$$\sigma_B(N-1) = \frac{\delta p_F(1 - \sigma_B(0))}{c} < 1$$
$$\downarrow$$
$$\frac{\delta p_F - c}{\delta p_F} < \sigma_B(0).$$

I have already verified that this inequality holds. Next, because $\sigma_B(N-1) < 1$, it must be the case that

$$\sigma_B(N-2) = \frac{\delta p_F(\sigma_B(N-1) - \sigma_B(0))}{c} < \sigma_B(N-1).$$

Consequently, $\{\sigma_B(I)\}_{I=0}^{N-1}$ is increasing and bounded by 1. I have already shown that $\sigma_B(0)$ is positive for all $N \geq 1$, so this completes the first part of the proof.

2. First, note that given the strategies in state $I = N$, $\{\sigma_B(I)\}_{I=0}^{N-1}$ is uniquely determined. Furthermore, for buyers to mix in states $I < N$, buyers must be kept indifferent. Consequently, $\sigma_S(I)$, in states $I < N$ and in an equilibrium, is bounded below and above by p_F. Thus, to verify that the equilibrium in Proposition 3 constitutes an efficient equilibrium (is constrained efficient), I need only demonstrate that there cannot be an equilibrium in which the seller provides high quality in state $I = N$.

   For the seller to provide high quality in state $I = N$, the seller would have to be indifferent in that state. For the seller to be indifferent in state $I = N$, the following condition must hold:

   $$\delta p_F(\sigma_B(N) - \sigma_B(0)) = c\sigma_B(N).$$

   In state $I = N - 1$, the seller is indifferent if

   $$\delta p_F(\sigma_B(N) - \sigma_B(0)) = c\sigma_B(N-1).$$

   Note that the seller must be indifferent in state $I = N-1$ if she is indifferent in state $I = N$. If she preferred low quality in state $I = N - 1$, buyers would not purchase in state $I = N - 1$, which in turn would imply that $\delta p_F(\sigma_B(N) - \sigma_B(0)) < 0$. If $\delta p_F(\sigma_B(N) - \sigma_B(0)) < 0$, the seller cannot be indifferent in state $I = N$ because $\delta p_F(\sigma_B(N) - \sigma_B(0)) < 0$ would imply that $\delta p_F(\sigma_B(N) - \sigma_B(0)) < c\sigma_B(N)$. Furthermore, because the RHS in the two conditions above are identical, they can hold simultaneously only if $\sigma_B(N-1) = \sigma_B(N)$. Using $\sigma_B(N-1) = \sigma_B(N)$, the seller is indifferent in state $N-2$ if

   $$\delta p_F(\sigma_B(N) - \sigma_B(0)) = c\sigma_B(N-2).$$

   Note again that the seller has to be indifferent in state $I = N-2$ if she is indifferent in state $I = N$. If she preferred low quality, it would imply that $\delta p_F(\sigma_B(N) - \sigma_B(0)) <$

0, which in turn would imply that $\delta p_F(\sigma_B(N) - \sigma_B(0)) < c\sigma_B(N)$. Note also that $\sigma_B(N-2)$ must be larger or equal to $\sigma_B(N)$. Continuing this process backwards ultimately results in $sigma_B(N) < \sigma_B(0)$. If $\sigma_B(N) < \sigma_B(0)$, there is no $\sigma_B(N)$ that can make the seller indifferent in state $I = N$. As such, there cannot be an equilibrium where the seller is indifferent in state $I = N$.

□

The following corollary establishes a result regarding the limit properties of the model when the price is exogenously fixed.

Corollary 2. ***Convergence to Stackelberg payoffs****: Consider some exogenously fixed price profile $\boldsymbol{p} = (p_F, p_F, ..., p_F)$. Then normalized equilibrium payoffs in any state $N - i$ converge to $p_F - c$ as $N \longrightarrow \infty$ for some $i \in \{1, 2, ..., N-2, N-1\}$. Furthermore, normalized equilibrium payoffs in state $I = 0$ converge to $p_F - c$ as $N \longrightarrow \infty$ and $\delta \longrightarrow 1$.*

Proof. Note first that $\lim_{N\longrightarrow\infty} \sigma_B(0) = \frac{\delta p_F - c}{\delta p_F}$. Using this, I can show that $\sigma_B(N-i)$ converges to 1 as $N \longrightarrow \infty$ for any positive integer $i < N$.
From the equation (8),

$$\delta p_F(\sigma_B(N-i+1) - \sigma_B(0)) = c\sigma_B(N-i)$$

must hold in all states $I < N$. Taking the limit as $N \longrightarrow \infty$ while keeping i constant results in

$$\delta p_F(\lim_{N\longrightarrow\infty} \sigma_B(N-i+1) - \frac{\delta p_F - c}{\delta p_F}) = c \lim_{N\longrightarrow\infty} \sigma_B(N-i),$$

which will hold only for $\lim_{N\longrightarrow\infty} \sigma_B(N-i+1) = \lim_{N\longrightarrow\infty} \sigma_B(N-i) = 1$.

Because the seller is indifferent in any state $I < N$ and prefers low quality in state N, the discounted payoffs $V_S^*(N-i, \boldsymbol{\sigma}_B, \boldsymbol{p})$ for any positive integer $i \leq N$, in the limit, are given by

$$\lim_{N\longrightarrow\infty} V_S^*(N-i, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p_F + \delta \lim_{N\longrightarrow\infty} V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p_F + \frac{\delta p_F - c}{1-\delta}.$$

Normalizing this by multiplying by $1-\delta$ results in

$$(1-\delta) \lim_{N\longrightarrow\infty} V_S^*(N-i, \boldsymbol{\sigma}_B, \boldsymbol{p}) = p_F - c,$$

which is equal to the per-period Stackelberg payoffs. In state $I = 0$, the seller's normalized payoffs are $\frac{\delta p_F - c}{\delta}$, which converges to $p_F - c$ when $\delta \longrightarrow 1$. This completes the proof. □

Online Appendix:

Endogenous Prices in Markets with Reputational Concerns

Magnus Våge Knutsen

The material contained herein is supplementary to the article "Endogenous Prices in Markets with Reputational Concerns". This online appendix contains four sections: A, B, C and D. In section A, I relax the assumption regarding the cost of quality. Instead of assuming that the seller pays the cost of quality only if the buyer purchases, I assume that the seller may pay any share of the cost regardless of a buyer's purchasing decision. In section B, I show that equilibria characterized in Propositions 1 and 2 exist in a model with only strategic sellers who vary with respect to the cost of quality. In section C, I consider a model where both the seller and the buyers post a price and one price is then implemented at random with a given probability. In section D I consider a perturbed version of the baseline model where noise is added to the cost of quality.

A An alternative cost structure

In this section I show that the results in Propositions 1 and 2 are robust to general assumptions regarding the cost of quality. Specifically, I assume here that the seller pays a share $\alpha \in [0,1]$ of the costs regardless of the buyer's purchasing decision. The stage-game payoff of the seller then becomes $u_S(q,b) = pb - q(\alpha c + (1-\alpha)cb)$.

Given the more general cost structure, the condition in equation 8 becomes:

$$\delta(\sigma_B(I^+)p(I^+) - \sigma_B(0)p(0)) = \alpha c + \sigma_B(I)c(1-\alpha). \tag{A.1}$$

Using equation A.1, I characterize a stationary PBE with seller-posted prices and with buyer-posted prices.

Proposition A.1. ***Seller-posted prices:*** *Assume that $\mu > 0$, $\delta > c$ and $\alpha \in [0,1]$. Then, there is some integer $\overline{N} < \infty$ such that the following constitutes a stationary PBE for any $N \geq \overline{N}$:*

Price proposals: *$p(N) = \mu(N)$ for both seller types and $p(I) = 1 \;\forall\; I \in \{0,1,...,N-1\}$.*

Quality choice: *$\sigma_S(N) = 0$ if $p(N) \leq \phi(p(N),N)$, $\sigma_S(N) = \frac{p(N)-\phi(p(N),N)}{1-\phi(p(N),N)}$ otherwise, and $\sigma_S(I) = p(I) \;\forall\; I \in \{0,1,...,N-1\}$.*

Beliefs: *$\mu(I) = 0 \;\forall\; I \in \{0,1,...,N-1\}$ and $\mu(N) = \frac{\mu(N+1)}{\mu N+1}$; $\phi(p(I),I) = 0 \;\forall\; I \in \{0,1,...,N-1\}$, $\phi(p(N),N) = \mu(N)$ if $p(N) = \mu(N)$ and $\phi(p(N),N) \leq \mu(N)$ otherwise.*

Purchasing decision: *$\sigma_B(N) = 1$ if $p(N) \leq \phi(p(N),N)$ and $\sigma_B(N) = \sigma_B(N-1)$ otherwise; $\sigma_B(I)$ for $I \in \{1,2,...,N-1\}$ is uniquely determined from equation A.1 for $p(I) \leq 1$ and $\sigma_B(I) = 0$ otherwise, and*

$$\sigma_B(0) = \begin{cases} \frac{p(N)-\frac{\alpha c}{\delta}k(N)}{k(N+1)} & \text{if } p(0) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

where $k(x) = \frac{\delta}{\delta - c(1-\alpha)}\left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^x\right)$.

Proof. Consider first the committed seller. By posting a price $p(N) < \frac{\mu(N+1)}{\mu N+1}$ in state N, the committed seller would earn strictly lower payoffs: The buyer either purchases with certainty if $p(N) \leq \phi(p(N),N)$ or mixes according to $\sigma_B(N-1)$ if $p(N) > \phi(p(N),N)$. Consequently, posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ results in higher payoffs than posting any lower price does. By posting a price, $p(N) > \frac{\mu(N+1)}{\mu N+1}$ in state N, the buyer will play as if in state $N-1$. Posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ is a best response if $\frac{\mu(N+1)}{\mu N+1} \geq \sigma_B(N-1)$. I show later in the proof that this inequality must hold. Thus, what remains to be shown is that buyer and seller strategies constitute mutual best responses given beliefs and the price proposals of the committed seller.

The rest of the proof proceeds as follows: I first verify that the strategies and beliefs in state $I = N$ can be a part of an equilibrium. I then consider states $I < N$.

Consider the strategies in state $I = N$. Conditional on $\sigma_S(I) = p(I) = 1 \ \forall \ I < N$, the probability of a strategic seller being in state N is $\lambda(N) = \frac{1}{1+N}$. Using $\lambda(N) = \frac{1}{1+N}$ together with equation 2, I find that the belief on the committed type in state $I = N$ is $\frac{\mu(N+1)}{\mu N+1}$. Conditional on $\sigma_S(N) = 0$, $\sigma_B(N) = 1$, and $\phi(I, p)$, I now verify that the best response of the seller is to set the price at the buyers' reservation price, i.e., $p(N) = \frac{\mu(N+1)}{\mu N+1}$, which is the same price as the committed seller posts in this state. First, by posting a price $p(N) < \frac{\mu(N+1)}{\mu N+1}$ in state N, the seller will earn strictly lower payoffs. Second, by posting a price, $p(N) > \frac{\mu(N+1)}{\mu N+1}$ in state N, the buyers will play as if in state $N-1$ (these strategies are derived below) and the seller plays a mix such that the buyers are indifferent. Posting $p(N) = \frac{\mu(N+1)}{\mu N+1}$ is a best response if

$$\mu(N) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p} \geq \sigma_B(N-1) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}),$$

which holds if $\frac{\mu(N+1)}{\mu N+1} \geq \sigma_B(N-1)$. I show later in the proof that the inequality must hold.

Next, because the buyers are indifferent if $p(N) = \frac{\mu(N+1)}{\mu N+1}$, $\sigma_B(N) = 1$ is a best response. Furthermore, given the off-path beliefs and the strategy of the seller following an off-path price proposal, the strategy of buyers constitutes a best response. For $\sigma_S(N) = 0$ to be a best response, the following condition must be satisfied:

$$\delta(V_S^*(N, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) = \delta(p(N) - \sigma_B(0)) \leq c.$$

The first equality in the condition above uses Lemma 2. Rewriting this condition, I find that low quality is a best response in $I = N$ if $\sigma_B(0) \geq \frac{\delta p(N) - c}{\delta}$. I show later in the proof that this holds; see the second step below.

Next, I consider the strategies for $I < N$. I start by verifying that $p(I) = 1$ in all states $I < N$ constitutes a best response. I start by fixing some commonly known price sequence $\boldsymbol{p}'$ and a strategy $\boldsymbol{\sigma}'_B$ of buyers such that the seller is indifferent in all states and I let the seller play according to $\sigma_S(I) = p(I) \ \forall \ I \in \{0, 1, ..., N-1\}$ and $\sigma_S(N) = 0$. Then, conditional on the price sequence $\boldsymbol{p}'$, $\boldsymbol{\sigma}_S$ and $\boldsymbol{\sigma}'_B$ are mutual best responses. Because $\boldsymbol{p}'$ is common knowledge, $\boldsymbol{\sigma}'_B$ is a function of $\boldsymbol{p}'$. It is, however, not a function of the price posted in any one period. That is, if the seller posts any other price, there is no reason for buyers to deviate: Because the seller's continuation payoffs are unchanged, the mix that makes the seller indifferent in that particular period must be unchanged. Thus, the seller posts the price that maximizes payoffs conditional on $\boldsymbol{\sigma}'_B$ and $\boldsymbol{p}'$, and because $\boldsymbol{\sigma}'_B$ is independent of this price, payoffs are maximized by setting the price to 1. Now, because $\boldsymbol{p}'$ in equilibrium must correspond to optimal price proposals, $\boldsymbol{p}' = \{1, 1, 1, ..., 1\}$.

Using $p(I) = 1$ for all states $I < N$, I derive the equilibrium strategy of the buyers. First, note that for $\sigma_S(I) = p(I)$, buyers are indifferent in states $I < N$. As such, any mix is a best response. To determine $\sigma_B(I)$ for $I < N$ that also makes the seller indifferent, I use equation A.1 and consider any integer $I \in [0, N)$. First, note that for $\alpha = 1$, it follows from equation A.1 that $\delta(\sigma_B(I)p(I) - \sigma_B(0)p(0)) = c$ for all $I \in \{1, ..., N\}$. Given $p(I) = 1$ for all $I \in \{0, ..., N-1\}$, this condition can hold only if $\sigma_B(I) = p(N)$ for all $I \in \{0, ..., N-1\}$, in which case $\sigma_B(0)$ follows directly from equation A.1 in any state: $\sigma_B(0) = \frac{\delta p(N) - c}{\delta}$.[38]

When $\alpha \in (0, 1)$, the argument resembles that of $\alpha = 0$. If the seller is indifferent between high and low quality in state I, the following condition must hold:

$$\delta(\sigma_B(I+1) - \sigma_B(0)) = \alpha c + (1-\alpha)c\sigma_B(I)$$
$$\updownarrow$$
$$\sigma_B(I) = \frac{\delta(\sigma_B(I+1) - \sigma_B(0)) - \alpha c}{(1-\alpha)c}.$$

This implies that $\sigma_S(I) = p(I)$ is a best response if there is a $\sigma_B(n)$ that makes the seller indifferent in states $I < N$. At $I = 0$, the condition is slightly different because this is the only state, apart from N, that the seller can remain in from one period to the next. In state $I = 0$, the condition in equation (A.1) becomes

$$\delta(\sigma_B(1) - \sigma_B(0)) = \alpha c + \sigma_B(0)c(1-\alpha)$$
$$\updownarrow$$
$$\sigma_B(0) = \frac{\delta\sigma_B(1) - \alpha c}{1 + c(1-\alpha)}.$$

To determine $\sigma_B(0)$ as well as the sequence $\{\sigma_B(I)\}_{I=1}^{N-1}$, I start at $N-1$, taking $\sigma_B(0)$ as given, and then applying backward induction. This process results in equation A.2.

Next, I verify that the sequence $\{\sigma_B(I)\}_{I=0}^{N-1}$ is well defined (bounded within the unit

[38] Setting $\alpha = 1$ in equation A.1 will result in the same solution.

interval). First, I show that there is a $\hat{N}$ such that $0 < \sigma_B(0) < 1$ for all $N \geq \overline{N}$. I do so in three steps. First, I show that $\lim_{N \longrightarrow \infty} \sigma_B(0) \in (0,1)$. Second, I show that $\sigma_B(0)$ is bounded below by a strictly increasing sequence that is larger than 0 for some $\overline{N}$, and third, I show that $\sigma_B(0)$ is bounded above by a monotone sequence that is bounded below 1.

First step: Taking the limit of equation (A.2) as $N \longrightarrow \infty$ results in the following:

$$\lim_{N \longrightarrow \infty} \sigma_B(0) = \frac{\delta - c}{\delta}$$

which is bounded by the unit interval for all $\delta > c > 0$.

Second step: To show that there is a $\overline{N}$ such that $0 < \sigma_B(0)$ for all $N \geq \hat{N}$, I show that $\sigma_B(0)$ is larger than $\frac{\delta p(N) - c}{\delta}$, where $p(N) = \frac{\mu(N+1)}{\mu N + 1}$. Because $\frac{\delta p(N) - c}{\delta}$ is monotone and converges to a positive number, this is sufficient to show that there is a $\hat{N}$ such that $\sigma_B(0) > 0$ for all $N \geq \overline{N}$. For $\sigma_B(0) \geq \frac{\delta p(N) - c}{\delta}$ the following condition must hold:

$$\sigma_B(0) = \frac{p(N) - \frac{\alpha c}{\delta} k(N)}{k(N+1)} \geq \frac{\delta p(N) - c}{\delta}.$$

By using the definition of $k(N)$, this can be rewritten to

$$\frac{\delta p(N) - c(1-\alpha)p(N) - \alpha c \left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^N\right)}{\delta \left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^{N+1}\right)} \geq \frac{\delta p(N) - c}{\delta}.$$

Because $\left(\frac{c(1-\alpha)}{\delta}\right)^n \in [0,1]$ and $p(N) < 1$, this weak inequality will always hold. Thus, I have verified that $\sigma_B(0) \geq \frac{\delta p(N) - c}{\delta}$. Note that because $p(N)$ and $1 - \left(\frac{c(1-\alpha)}{\delta}\right)^n$ converge to 1, there will always exist an integer $\overline{N} < \infty$ such that $\sigma_B(0) \geq 0$ if $\delta > c$.

Third step: Next, I show that $\sigma_B(0)$ is bounded above by $\frac{1 - \frac{\alpha c}{\delta} k(N)}{k(N+1)}$, where $\frac{1 - \frac{\alpha c}{\delta} k(N)}{k(N+1)}$ is bounded by 1 because $k(x) \geq 1$ for all $x > 1$. For $\sigma_B(0)$ to be bounded by $\frac{1 - \frac{\alpha c}{\delta} k(N)}{k(N+1)}$, the following condition must hold:

$$\frac{1 - \frac{\alpha c}{\delta} k(N)}{k(N+1)} \geq \frac{p(N) - \frac{\alpha c}{\delta} k(N)}{k(N+1)}$$

$$\downarrow$$

$$1 \geq p(N).$$

This clearly holds for all N.

Finally, I show that $\{\sigma_B(I)\}_{I=0}^{N-1}$ is an increasing sequence that is bounded below by $\sigma_B(0)$ and above by $p(N)$. I start at $N-1$. Note that $\sigma_B(N)p(N) = p(N)$ and $\sigma_B(N-1)p(N-1) = \sigma_B(N-1)$ in equilibrium. Consequently, $\sigma_B(N)p(N) = p(N) > \sigma_B(N-1)$ if

$$\sigma_B(N-1) = \frac{\delta(p(N) - \sigma_B(0)) - \alpha c}{(1-\alpha)c} < p(N)$$
$$\downarrow$$
$$\delta\sigma_B(0) > p(N)(\delta - c(1-\alpha)) - \alpha c.$$

By replacing $\sigma_B(0)$, I can rewrite this to

$$\frac{p(N)(\delta - c(1-\alpha)) - \alpha c\left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^N\right)}{\left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^{N+1}\right)} > p(N)(\delta - c(1-\alpha)) - \alpha c.$$

Because $\left(\frac{c(1-\alpha)}{\delta}\right)^N \in [0,1)\ \forall\, \alpha \in [0,1]$, this inequality must hold. As such, $\sigma_B(N-1) < p(N)$, where $p(N) = \frac{\mu(N+1)}{\mu N+1}$. Next, because $\sigma_B(N-1) < p(N)$, it must be the case that

$$\sigma_B(N-2) = \frac{\delta(\sigma_B(N-1) - \sigma_B(0)) - \alpha c}{(1-\alpha)c} < \sigma_B(N-1).$$

Consequently, $\{\sigma_B(n)\}_{n=0}^{N-1}$ is increasing and bounded by $p(N)$. I have already shown that $\sigma_B(0)$ is positive for all $N \geq \overline{N}$, so this completes the proof.

□

Proposition A.1 demonstrates that the features of the equilibrium characterized in Proposition 1 are robust to a more general assumption regarding the cost of quality. A couple of things warrant a comment. First, for a given prior, the equilibrium described in Proposition A.1 exists only if N is large enough. The reason is that the seller posts a price of 1 and provides high quality with certainty in states $I < N$. Consequently, for state $I = N$ to be sufficiently "rare" to ensure that posterior beliefs are such that $\sigma_B(0) = \frac{p(N) - \frac{\alpha c}{\delta}k(N)}{k(N+1)} > 0$, the observable history must be sufficiently long.

Second, for $\alpha < 1$, the equilibrium described in Proposition A.1 constitutes a unique efficient equilibrium.[39] For $\alpha = 1$, however, the equilibrium described in Proposition A.1 is neither unique nor an efficient equilibrium. Specifically, for $\alpha = 1$ there is a set of equilibria where the seller provides high quality with certainty in state $I = N$. In one of these equilibria, buyers also purchase with certainty in state $I = N$. Because these equilibria exist only in the knife-edge case where $\alpha = 1$, they are of little interest. Consequently, I do not provide a characterization of them here.

The next result concerns equilibria when $\alpha \in [0, 1]$ and buyers post prices.

Proposition A.2. ***Buyer-posted prices:*** *For $\alpha \in [0, 1]$, $\mu \geq 0$, $\delta > c$, and $\mu \leq \frac{\delta - c}{\delta}$ the following constitutes a stationary PBE for any $N \geq 1$:*

Price proposals: $p(I) = p' \in [\frac{c}{\delta}, 1 - \mu]$ $\forall$ $I \in \mathbf{I}$ *and* $p(0) = \frac{\delta p' - c}{\delta}$.

Quality choice: $\sigma_S(I) = p(I)$ *for all* $I \in \{0, ..., N - 1\}$; $\sigma_S(N) = 1$ *if* $p(N) \in [p', 1]$, $\sigma_S(N) = \frac{p(N) - \mu}{1 - \mu}$ *if* $p(N) \in [\mu, p')$, *and* $\sigma_S(N) = 0$ *otherwise.*

Beliefs: $\mu(I) = 0$ $\forall$ $I \in \{0, 1, ..., N - 1\}$ *and* $\mu(N) = \mu$.

Purchasing decision: $\sigma_B(I) = 1$ *for all* $I \in \{0, ..., N\}$ *and* $p(N) \in [0, 1]$.

The proof is identical to the proof of Proposition 2 and is therefore omitted. For $\alpha < 1$, Proposition A.2 gives a complete characterization of efficient equilibria with buyer-posted prices. However, when $\alpha = 1$, there are other efficient equilibria. For example, there is an equilibrium where buyers propose a price of $1 - \mu$ and purchase with certainty in state $I > 0$, and play a mix in state $I = 0$, and the seller provides high quality in state $I = N$ if $p(N) \geq 1 - \mu$, and plays a mixed strategy in other states. Again, because these equilibria exist only in the knife-edge case where $\alpha = 1$, they are of little interest. Consequently, I do not provide a characterization of them here.

[39] Proof is omitted because the argument is identical to that of $\alpha = 0$.

B A model without a committed seller

Reputation models with commitment types tend to lack a clear link to economic fundamentals. In the context of my model, showing that the behavior of the committed seller can be rationalized is of particular interest. The reason is that the committed seller in my model is somewhat of a hybrid. That is, the committed seller is committed to high quality, but she sets the price strategically. In this section, I show that the behavior of the committed seller can also arise in a model where types differ with respect to the cost of quality and where cost is private information. In this model, commitment types arise endogenously as a feature of the equilibrium rather than by assumption.

Consider the model as presented in section 2, but instead of a strategic and a committed seller type, there are two strategic seller types who differ only with respect to the cost of quality. Consider the type space $\theta \in \{H, L\}$ and let c_H and c_L denote the cost of quality for the H type and the L type, respectively. Furthermore, let $c_L > c_H > 0$ and μ_H denote the prior belief about H types. I let $\sigma_\theta(I)$ and $p_\theta(I)$ denote seller type θ's strategies, and $\mu_H(I)$ denote the posterior on the H type conditional on state I, and let $\phi_H(I, p)$ denote the posterior on the H type conditional on state I *and* the price proposal. Finally, I let $p(I)$ denote the price that the buyer observes. For the sake of generality, I maintain the generalization with respect to the cost of quality from section A of the online appendix. Under these new assumptions, I provide the following result under the assumption that the seller posts price.[40]

Proposition A.3. ***Seller-posted prices:*** *For any $\alpha \in [0, 1]$, $\mu_H \in (0, 1)$ and $\delta > c_L$, there is a positive integer $\hat{\hat{N}} < \infty$ and a $c_H > 0$ such that the following constitutes a stationary PBE for any $N \geq \hat{\hat{N}}$:*

Price proposals: *$p_H(N) = \mu_H(N)$ and $p_H(I) = 1 \; \forall \; I \in \{0, 1, ..., N-1\}$; $p_L(N) = \mu_H(N)$ and $p_L(I) = 1 \; \forall \; I \in \{0, 1, ..., N-1\}$.*

Quality choice: *$\sigma_H(I) = 1 \; \forall \; I \in \mathbf{I}$; $\sigma_L(N) = 0$ if $p(N) \leq \phi_H(p(N), N)$, $\sigma_L(N) = \frac{p(N) - \phi_H(p(N), N)}{1 - \phi_H(p(N), N)}$, $\sigma_L(I) = p_L(I) \; \forall \; I \in \{0, 1, ..., N-1\}$.*

Beliefs: *$\mu_H(I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$ and $\mu_H(N) = \frac{\mu_H(N+1)}{\mu_H N + 1}$; $\phi(p(I), I) = 0 \; \forall$ $I \in \{0, 1, ..., N-1\}$, $\phi_H(p(N), N) = \mu_H(N)$ if $p(N) = \mu_H(N)$ and $\phi_H(p(N), N) \leq \mu_H(N)$ otherwise.*

Purchasing decision: *$\sigma_B(N) = 1$ if $p(N) \leq \phi_H(p(N), N)$ and $\sigma_B(N) = \sigma_B(N-1)$ otherwise, $\sigma_B(I)$ for $I \in \{1, 2, ..., N-1\}$ is uniquely determined from equation A.1 if*

[40] Note that a similar result can be obtained by letting types vary with respect to their discount factors.

$p(I) \leq 1$ *and* $\sigma_B(1) = 0$ *otherwise, and*

$$\sigma_B(0) = \begin{cases} \frac{p(N) - \frac{\alpha c}{\delta} k(N)}{k(N+1)} & \text{if } p(0) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.3}$$

and $k(x) = \frac{\delta}{\delta - c(1-\alpha)} \left(1 - \left(\frac{c(1-\alpha)}{\delta}\right)^x\right)$.

Proof. In Proposition A.1, I established that a strategic seller will provide high quality in states $I < N$ given that there is some seller type that always provides high quality. Consequently, the strategy of the L type constitutes a best response to the buyers' strategy, given that the H type plays as the committed seller. Furthermore, the buyers' strategy is a best response to the seller's strategy, again given that there is a seller type that always provides high quality. Also, from A.1, there exists an integer $\hat{\hat{N}} < \infty$ such that the sequence $\{\sigma_B(I)\}_{I=0}^{N-1}$ is well defined. I need to verify only that it is indeed optimal for the H type to always provide high quality and post a price equal to $\mu_H(N)$ for some $c_H > 0$.

I first consider the quality choice. The strategy of buyers is such that equation 8 holds with equality in any state $I < N$ for the L type, which must imply that the H type strictly prefers to provide high quality because $c^L > c^H$. Furthermore, the mix played by the buyers in any such state is independent of the price. As such, $p(I) = 1$ is optimal. Note that this is true only due to $\mu(I) = 0 \ \forall \ I \in \{0, 1, ..., N-1\}$, which still remains to be shown. For this belief to be an equilibrium belief, $\sigma_H(N) = 1$ must be optimal. This is the case if the H type weakly prefers high quality in state $I = N$. Using equation 8 along with $\sigma_B^*(0)$, I find that this holds if

$$\delta \left(p_\theta(N) - \frac{p_\theta(N) - \frac{\alpha c_L}{\delta} k(N)}{k(N+1)}\right) \geq c_H.$$

Because the LHS is strictly positive, there is a $c^H > 0$ such that the inequality holds.

Finally, consider the price proposals of the H type. Following any price proposal $p_H(N) \neq \frac{\mu_H + N\mu_H}{N\mu_H + 1}$, buyers play according to $\sigma_B(N-1)$. Furthermore, given $\sigma_B(N-1)$, the optimal deviation is to post a price of 1. Thus, given the continuation strategy of the H type, $p_H(N) = \frac{\mu_H + N\mu_H}{N\mu_H + 1}$ is a best response if

$$p(N) - c_H \geq \sigma_B(N-1) - \alpha c_H - (1-\alpha) c_H \sigma_B(N-1)$$
$$\Updownarrow$$
$$p(N) - \sigma_B(N-1) \geq c_H(1-\alpha)(1 - \sigma_B(N-1)).$$

If $\alpha = 1$, $p(N) = \sigma_B(N-1)$ (see the proof of Proposition A.1) and so the inequality holds. If $\alpha < 1$, $p(N) > \sigma_B(N-1)$ (see the proof of Proposition A.1) and so there is always a $c_H > 0$ such that this inequality holds. This completes the proof.

□

First, note that the equilibrium behavior of the H type coincides with the behavior of the committed seller while the L type plays as the strategic seller. The intuition behind this result is that the H type benefits from distinguishing herself from the L type. Given the strategy of the L type, the only way she can do so is by always providing high quality. Furthermore, the H type posts a price equal to the belief on her type conditional on the state, given the behavior of the L type. This implies that the H type does not signal her type by posting a lower price. As the proposition states, this holds under fairly flexible out-of-equilibrium beliefs: I allow buyers to assign a higher belief on the H type following any price proposal below the equilibrium price proposal.

The result in Proposition A.3 highlights a nice feature of this version of my model: Cyclical and divergent reputation dynamics can be contained in the same equilibrium in a model without commitment types. The L type displays cyclical behavior by building reputation only to exploit it, while the H type always provides high quality. This result contrasts with, e.g., Board and Meyer-ter Vehn (2013) and Dilmé (2019), in which equilibria display either cyclical or divergent reputation dynamics depending on the news process, which is always held fixed in my analysis.

Next, I consider equilibria when buyers post price. In the proposition below, $p(I)$ denotes price proposals by the buyer.

Proposition A.4. ***Buyer-posted prices:*** *For any $\alpha \in [0,1)$, $\mu_H \geq 0$, $\delta > c_L$, and $\mu_H \leq \frac{\delta - c_L}{\delta}$ the following constitutes a stationary PBE for any $N \geq 1$:*

Price proposals: $p(I) = p' \in [\frac{c}{\delta}, 1-\mu]\ \forall\ I \in \{1, ..., N\}$ *and* $p(0) = \frac{\delta p' - c_L}{\delta}$.

Quality choice: $\sigma_H(I) = 1$ *for all* $I \in \{0, ..., N\}$ *and* $p(I) \in [0,1]$*;* $\sigma_L(I) = p(I)$ *for all* $I \in \{0, ..., N-1\}$*,* $\sigma_L(N) = 1$ *if* $p(N) \in [p', 1]$*,* $\sigma_L(N) = \frac{p(N) - \mu_H}{1-\mu_H}$ *if* $p(N) \in [\mu_H, p')$*, and* $\sigma_H(N) = 0$ *otherwise.*

Beliefs: $\mu_H(I) = 0\ \forall\ I \in \{0, 1, ..., N-1\}$ *and* $\mu_H(N) = \mu_H$.

Purchasing decision: $\sigma_B(I) = 1$ *for all* $I \in \{0, ..., N\}$ *and* $p(N) \in [0,1]$.

Proof. In Proposition 2 I established that a strategic seller is indifferent given the strategy of buyers. Consequently, the strategy of the L type constitutes a best response to the buyers' strategy. If the L type is indifferent, it follows that high quality must be a strict best response for the H type for any price proposals. Finally, as in Proposition 2, the prior on the seller type who always provides high quality must be sufficiently small ($\mu_H \leq \frac{\delta - c_L}{\delta}$) for buyers to propose a strictly positive price. □

C Stochastic bargaining power

In this section I demonstrate that the results in Propositions 1 and 2 are somewhat robust in the sense that they do not require that either the seller or the buyers have full influence over price. In what follows, I assume that both the seller and the buyers post a price. Each proposal is then observed by both players and implemented with a certain probability. I let β denote the probability that the seller's price proposal is implemented.[41] Then $\beta = 1$ and $\beta = 0$ correspond to the two cases covered in the article.

I let $p_S(I)$ and $p_B(I)$ denote price proposals by the seller and the buyers, respectively, I let $p_R(I)$ denote the realized price, I let $\overline{p}(I) = \beta p_S(I) + (1-\beta)p_B(I)$ denote the expected price, and I let $\overline{\boldsymbol{p}}$ denote the vector of expected prices. I assume that buyers observe the implemented price as well as the proposed price by the seller. Consequently, in state $I = N$ buyers may still update beliefs regarding the seller's type after observing the price proposal even if the seller's price is not implemented. Furthermore, I let $V_S(I, \boldsymbol{\sigma}_S, \boldsymbol{\sigma}_B, \overline{\boldsymbol{p}}, \beta)$ and $u_B(\beta)$ denote payoffs to the seller and the buyers, respectively, as functions of β.

Propositions A.5 and A.6 (below) focus on the cases where both sides of the market have some influence over price ($\beta \in (0,1)$). Proposition A.5 focuses on the type of equilibrium characterized in Proposition 1 where the seller builds reputation in states $I < N$ only to exploit it in state $I = N$. This equilibrium exists for all $\beta \in (0,1)$. For this case, I focus on an equilibrium where the seller and the buyers post the same price in states $I < N$, which greatly simplifies this analysis because β will affect the expected price only in state $I = N$.[42]

Proposition A.5. *Assume that $\mu > 0$ and $\delta > c$ and $\beta \in (0,1)$. Then, for any $N \geq 1$, the following constitutes a stationary PBE:*

Price proposals: *$p_S(N) = \mu(N)$ for both seller types, $p_B(N) = 0$, and $p_i(I) = 1$ $\forall$ $I \in \{0,1,...,N-1\}$ and $i \in \{B,S\}$.*

Quality choice: *$\sigma_S(N) = 0$ if $p_R(N) \leq \phi(p_S(N), N)$, $\sigma_S(N) = \frac{p_R(N) - \phi(p_S(N),N)}{1-\phi(p_S(N),N)}$ otherwise, and $\sigma_S(I) = p_R(I)$ $\forall$ $I \in \{0,1,...,N-1\}$.*

Beliefs: *$\mu(I) = 0$ $\forall$ $I \in \{0,1,...,N-1\}$ and $\mu(N) = \frac{\mu(N+1)}{\mu N+1}$; $\phi(p_S(I), I) = 0$ $\forall$ $I \in \{0,1,...,N-1\}$, $\phi(p_S(N), N) = \mu(N)$ if $p_S(N) = \mu(N)$ and $\phi(p_S(N), N) \leq \mu(N)$*

[41] We can think about β as a parameter that captures market conditions or some bargaining process that is not explicitly modeled.

[42] There are also equilibria where $p_B(I) < 1$ in states $I \in \{0,1,...,N-1\}$. While these equilibria differ quantitatively from the one characterized in Proposition A.6, they share the same qualitative features: The seller builds her reputation in states $I \in \{0,1,...,N-1\}$, and exploits it in state $I = N$. The key quantitative difference is that reputation-building will take longer if $p_B(I) < 1$ in some states because the price determines the mix played by the seller.

otherwise.

Purchasing decision: *$\sigma_B(N) = 1$ if $p_R(N) \leq \phi(p_S(N), N)$ and $\sigma_B(N) = \sigma_B(N-1)$ otherwise, $\sigma_B(I) = \frac{\delta(\overline{p}(I^+)\sigma_B(I^+)\sigma_B(0))}{c}$ $\forall$ $I \in \{1, 2, ..., N-1\}$ if $p(I) \leq 1$ and $\sigma_B(I) = 0$ otherwise, and*

$$\sigma_B(0) = \begin{cases} \frac{\beta p_S(N)}{k(N+1)} & \text{if } p(0) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.4}$$

where $k(x) = \frac{\delta}{\delta - c}\left(1 - \left(\frac{c}{\delta}\right)^x\right)$.

Proof. Consider first the committed seller. By Proposition 1 and its proof, it follows that $p(N) = \frac{\mu(N+1)}{\mu N+1}$ strictly dominates any other price proposals. Thus, what remains to be shown is that buyer and seller strategies constitute mutual best responses given beliefs and the price proposals of the committed seller.

Conditional on the expected price profile $\overline{\boldsymbol{p}} = (1, 1, ..., 1, \beta\mu(N))$, the strategies of the seller and the buyers constitute mutual best responses. This follows from Proposition 1 and its proof. Furthermore, the price proposals of the seller also constitute a best response. This also follows from Proposition 1 and its proof. Thus, I need to verify only that the price proposals of buyers constitute a best response. Given that $\sigma_S(I) = p_R(I)$ for $I < N$, buyers are indifferent between price proposals. As such, $p_B(I) = 1$ is for $I < N$ a best response. In state $I = N$, $\sigma_S(I) = 0$ regardless of the proposed (and implemented) price. As such, $p_B(N) = 0$ is a best response.

Finally, by the first and second steps in part 1 of the proof of Proposition 1, it follows that $\{\sigma_B(I)\}_{I=0}^{N-1}$ is well defined (bounded within the unit interval). This completes the proof. □

The equilibrium characterized in Proposition A.5 has characteristics similar to the one characterized in Proposition 1. In particular, $\sigma_B(I+1) > \sigma_B(I)$ for all $I \in \{0, 1, ..., N-1\}$, $\sigma_B(N) = 1$ and $\sigma_S(N) = 0$. The following result shows how payoffs and surplus vary with respect to β.

Corollary A.1. *The stationary PBE characterized in Proposition A.5 has the following properties:*

i) $\frac{\partial V_S^*(I, \boldsymbol{\sigma}_B, \boldsymbol{p}, \beta)}{\partial \beta} > 0 \; \forall \; I,$

ii) $\frac{\partial E[u_b(\beta)]}{\partial \beta} < 0,$ *and*

iii) social surplus is increasing in β.

Proof. I use the property that $V_S^*(I, \boldsymbol{\sigma}_B, \overline{\boldsymbol{p}}, \beta) = \sigma_B(I) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \overline{\boldsymbol{p}}, \beta)$ and that $V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}, \beta) = \frac{\sigma_B(0,)}{1-\delta}$. Then, by the definition of $\sigma_B(0,)$ from equation A.4, $\frac{\partial V_S^*(0,\boldsymbol{\sigma}_B,\overline{\boldsymbol{p}},\beta)}{\partial \beta} > 0$ and $\frac{\partial V_S^*(N,\boldsymbol{\sigma}_B,\boldsymbol{p},\beta)}{\partial \beta} > 0$ follow immediately. To complete the proof I need to show that $\frac{\partial V_S^*(I,\boldsymbol{\sigma}_B,\boldsymbol{p},\beta)}{\partial \beta} > 0$ for the histories $I \in \{1, 2, ..., N-1\}$. Because it is already established that $\frac{\partial V_S^*(0,\boldsymbol{\sigma}_B,\boldsymbol{p},\beta)}{\partial \beta} > 0$, it suffices to show that $\frac{\partial \sigma_B(I,\beta)}{\partial \beta} > 0 \; \forall \; I \in \{1, 2, ..., N-1\}$. To show this, consider equation (8) in state $I = 0$

$$\delta(\sigma_B(1) - \sigma_B(0)) = c\sigma_B(0).$$

Because $\sigma_B(0)$ is increasing in β, $\sigma_B(1)$ must also be increasing in β for the condition to hold. Furthermore, because both $\sigma_B(0)$ and $\sigma_B(1)$ are increasing in β, $\sigma_B(2)$ must be increasing in β for the condition to hold in state $I = 1$:

$$\delta(\sigma_B(2) - \sigma_B(0)) = c\sigma_B(1).$$

Clearly the same argument must hold for any $I \in \{1, 2, ..., N-1\}$.

Next, I consider the buyers. The expected payoff of buyers is given by:

$$E[u_b(\beta)] = \mu\left(\beta(1 - p_S(N)) + (1-\beta)\right) + (1-\mu)\lambda(N)(-\beta p_S(N)).$$

First, I can verify that $\mu(1 - p_S(N)) + (1-\mu)\lambda(N)(-p_S) = 0$, i.e., that the expected payoff to buyers when the seller posts price is zero. Replacing $p_S(N)$, I obtain the following:

$$\begin{aligned}
&\mu\left(1 - \frac{\mu}{\mu + (1-\mu)\lambda(N)}\right) + (1-\mu)\lambda(N)\left(-\frac{\mu}{\mu + (1-\mu)\lambda(N)}\right) \\
=&\mu\left(\frac{(1-\mu)\lambda(N)}{\mu + (1-\mu)\lambda(N)}\right) + (1-\mu)\lambda(N)\left(-\frac{\mu}{\mu + (1-\mu)\lambda(N)}\right) \\
=&0,
\end{aligned}$$

which is equal to zero. Consequently, the expected payoff to buyers simplifies to

$$E[u_b(\beta)] = (1-\beta)\mu$$

which is decreasing in β. Thus, $\frac{\partial E[u_b(\beta)]}{\partial \beta} < 0$.

Finally, the result regarding social surplus follows from $\frac{\partial \sigma_B(I)}{\partial \beta} > 0 \; \forall \; I \in \{0, 1, ..., N-1\}$, which I have already shown. This completes the proof. □

Consider first the effect of β on seller payoffs (part i). Increasing β will increase the payoffs to the seller in state $I = N$. Furthermore, because payoffs increase in state $I = N$, buyers must purchase at a higher rate in states $I < N$ for the seller to be indifferent in these states. That is, when the gain from having a good reputation increases, the costs of building reputation must increase as well. The effect of β on social surplus (part iii) follows directly from the property that a higher gain from a good reputation allows for higher costs of building a good reputation. When β increases, buyers can purchase at a higher rate in states $I < N$. Because the seller provides high quality with certainty in states $I < N$, this implies that the rate of mutually beneficial transactions will increase in β. Finally, the effect of β on buyer payoffs is negative (part iii). This is driven by outcomes in state $I = N$ only. When the buyers' price is implemented in state $I = N$, the buyer earns an expected payoff of $\mu(N) = \frac{\mu(N+1)}{\mu N+1}$. If the seller's price is implemented, the buyer earns an expected payoff of 0.

Proposition A.6 focuses on the case where β is relatively small, in which case there is an equilibrium that resembles the equilibrium characterized in Proposition 2 where the seller always provides high quality. For this case, I focus on an equilibrium where buyers post a price in state $I = N$ such that they are indifferent between this price and offering a price of 0 and earning an expected payoff of μ.[43]

Proposition A.6. *Assume that* $\mu \in [0, \frac{\delta-c}{\delta}]$, $\delta > c$ *and* $\beta \in (0, \frac{\delta(1-\mu)-c}{\delta(1-\mu)}]$. *Then, for any* $N \geq 1$, *the following constitutes a stationary PBE:*

Price proposals: $p_S(N) = 1$ *for both seller types,* $p_S(I) = 1 \; \forall \; I \in \{0, 1, ..., N-1\}$, $p_B(I) = 1 - \mu \; \forall \; I \in \{1, ..., N\}$ *and* $p_B(0) = \frac{\delta(1-\mu)(1-\beta)-c}{\delta(1-\beta)}$.

Quality choice: $\sigma_S(I) = p_R(I)$ *for all* $I \in \{0, ..., N-1\}$; $\sigma_S(N) = 1$ *if* $p_R(N) \in [1-\mu, 1]$, $\sigma_S(N) = \frac{p_R(N)-\mu}{1-\mu}$ *if* $p_R(N) \in [\mu, 1-\mu)$, *and* $\sigma_S(N) = 0$ *otherwise.*

Beliefs: $\mu(I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$ *and* $\mu(N) = \mu$; $\phi(p_S(I), I) = 0 \; \forall \; I \in \{0, 1, ..., N-1\}$ *and* $\phi(p_S(N), N) = \mu(N)$ *for any* $p_S(N)$.

Purchasing decision: $\sigma_B(I) = 1$ *for all* $I \in \{0, ..., N\}$ *and* $p(N) \in [0, 1]$.

Proof. First, note that the price proposals of the committed seller are optimal given on-path and off-path beliefs. Thus, what remains to be shown is that buyer and seller strategies constitute mutual best responses given beliefs and the price proposals of the committed seller.

Second, I show that the equilibrium can exist only if $\beta \in (0, \frac{\delta(1-\mu)-c}{\delta}]$. Note that $p_B(0) = \frac{\delta(1-\mu-\beta)-c}{\delta(1-\beta)} \geq 0$ if $\beta \in (0, \frac{\delta(1-\mu)-c}{\delta}]$. If $\beta > \frac{\delta(1-\mu)-c}{\delta}$, there is no $p_B(0) \geq 0$, given the price

[43] This is the equilibrium that exists for the widest range of βs.

posted by the seller and $p_B(N) = \frac{1-\mu-\beta}{1-\beta}$, that is sufficient to induce high quality in state $I = N$. Thus, for the equilibrium to exist, β must be sufficiently low ($\beta \in (0, \frac{\delta(1-\mu)-c}{\delta}]$).

Third, I verify that the strategy of the seller constitutes a best response given the strategy of buyers. Given the strategy of buyers, the seller maximizes payoffs by posting $p_S(I) = 1$. As such, $p_S(I) = 1$ is a best response. Furthermore, in any state $I \in \{0, ..., N\}$, the seller is indifferent between high and low quality if

$$\delta(\overline{p}(I^+) - \overline{p}(0)) = c.$$

Using $\overline{p}(I^+) = \beta + (1-\beta)(1-\mu)$ and $\overline{p}(0) = \beta + (1-\beta)\frac{\delta(1-\beta)(1-\mu)-c}{\delta(1-\beta)}$, it can be verified that the condition above holds with equality. Because the seller is indifferent in all states, any strategy will constitute a best response.

Fourth, I verify that the price proposals of buyers constitute a best response given the strategy of the seller. In state $I < N$, the seller plays a mix such that buyers are indifferent between purchasing and not, given any price proposal. Consequently, any price proposal constitutes a best response. In state $I = N$, the seller provides high quality with certainty if $p_B(N) \geq 1-\mu$. As such, $p_B(N) = 1-\mu$ yields higher payoffs to buyers than any $p_B(N) > 1-\mu$. Next, for any price $p_B(N) \in [\mu, 1-\mu)$, the seller plays a mix such that buyers are indifferent between purchasing and not purchasing. Not purchasing results in a payoff of 0. Thus, $p(N) = 1-\mu$ results in a higher payoff than any $p(N) \in [\mu, \frac{1-\mu-\beta}{1-\beta})$ if $1-(1-\mu) \geq 0$. This holds because $\mu > 0$. Finally, for any $p(N) < \mu$, the seller provides low quality with certainty. This implies that the highest expected payoff that buyers can earn from a price proposal $p(N) < \mu$ is $\mu(1-\beta)$ (by proposing $p(N) = 0$). Thus, $p(N) = 1-\mu$ results in a higher payoff than any $p(N) < \mu$ if $\beta(1-p_S(N)) + (1-\beta)(1-(1-\mu)) \geq \beta(1-p_S(N)) + (1-\beta)(\mu) \to \mu \geq \mu$.

Finally, I verify that the purchasing decisions constitute a best response. In states $I < N$, given any price proposal $p(I) \in [0,1]$ and $\sigma_S(I) = p(I)$, buyers are indifferent. As such, $\sigma_B(I) = 1$ is a best response. In state $I = N$, the strategy of the seller ensures that purchasing is a best response following any proposal. □

The equilibrium characterized in Proposition A.6 has characteristics similar to the one characterized in Proposition 2. The price proposals of buyers in states $I \in \{0, 1, ..., N-1\}$ and $I = 0$ are such that the expected difference in realized prices between the two states makes the seller exactly indifferent between high and low quality. For the expected realized price in state $I = 0$ to be low enough to ensure that the seller is indifferent, the probability that the price proposed by buyers will be implemented must be sufficiently large ($\beta \leq \frac{\delta(1-\mu)-c}{\delta(1-\mu)}$).

An implication of Propositions A.5 and A.6 and Corollary A.1 is that efficiency may be non-monotone in β. If the seller has little influence over price ($\beta \in (0, \frac{\delta(1-\mu)-c}{\delta(1-\mu)})$, there is an efficient equilibrium. However, once $\beta > \frac{\delta(1-\mu)-c}{\delta(1-\mu)}$, the equilibrium in Proposition A.6 no longer exists. In the equilibrium characterized in Proposition A.5, efficiency is increasing in β. Thus, if buyers have insufficient influence over price to sustain an efficient equilibrium under the looming threat of a low price following a choice of low quality, the most efficient outcome is ensured by letting the price proposed by the seller be implemented with certainty.

D Purifiable equilibria

In this sections I set up a perturbed version of the baseline model. I construct equilibria of this perturbed game, and consider the limit behavior in equilibria as the noise goes to zero. Specifically, suppose that in each period there is a shock εz to the cost of quality. I assume that z is i.i.d. across time according to a commonly known density with support $[-1, 1]$ and $E(z) = 0$. The cost of quality is then $c + \varepsilon z$. I assume that $\varepsilon \in (0, 1-c)$. Like in the baseline model, the cost is only incurred if the buyer makes a purchase. The shock is assumed to be privately observed by the seller prior to making any decisions. For the sake of simplicity I consider the case with one-period memory only ($N = 1$). Note that $N = 1$ is a special case and that the analysis of this game may be significantly more involved when $N > 1$.

Given the description of the perturbed game in the previous paragraph, an equilibrium of the game described in section 2 of the article (the unperturbed game) is purifiable if there is an equilibrium of the perturbed game that converges to the equilibrium of the unperturbed game as $\varepsilon \to 0$.[44]

D.1 A purifiable equilibrium with seller-posted prices

In this section I demonstrate that the equilibrium in Proposition 1 is purifiable for the case with one-period memory. I start by construction an equilibrium of the perturbed game and demonstrate that the equilibrium of the perturbed game converges to that of Proposition 1 when $\varepsilon \to 0$.

In the equilibrium of the perturbed game buyers mix in state $I = 0$ such that the seller strictly prefers high quality for $z < 1$ and is indifferent for $z = 1$. The mixed strategy will constitute a best response since the seller posts a price of 1. Posting a price of 1 in state $I = 0$ will be a best response for an appropriate specification of beliefs. In particular, for on-path price proposals, buyers do not update beliefs regarding z. For off-path price proposals, buyers infer with probability one that $z = 1$.

Equilibrium candidate of the perturbed game with seller-posted prices: Assume that $\mu > 0$, $\delta > c$ and $N = 1$, and consider the following equilibrium candidate of the perturbed game:

Price proposals: $p(1, z) = \mu(1)$ for both seller types and $p(0, z) = 1$.

Quality choice: $\sigma_S(1, z) = 0$ if $p(1, z) = \mu(1)$, $\sigma_S(1, z) = \sigma_S(0, z)$ otherwise; and

[44]See Bhaskar and Thomas (2018) for a discussion of purifiable equilibria in the context of bounded memory.

$\sigma_S(0,z) = 1$ if $p(0,z) = 1$ and for all z, $\sigma_S(0,z) = 1$ if $p(0,z) < 1$ and for all $z < 1$ and $\sigma_S(0,z) = p(0,z)$ if $p(0,z) < 1$ and for $z = 1$.

Beliefs about type: $\mu(0) = 0$ and $\mu(1) = \frac{2\mu}{\mu+1}$; $\phi(p(0),0) = 0$, $\phi(p(1),1) = \mu(1)$ if $p(1) = \mu(1)$ and $\phi(p(1),1) = 0$ otherwise.

Beliefs about z: $E[z|I = 0, p(0) = 1] = E[z] = 0$, $E[z|I = 0, p(0) < 1] = 1$, $E[z|I = 1, p(1) = \mu(1)] = E[z] = 0$, $E[z|I = 1, p(1) = 1] = E[z] = 0$ and $E[z|I = 1, p(1) \in [0,\mu(1)) \cup (\mu(1),1)] = 1$.

Purchasing decision: $\sigma_B(1) = 1$ if $p(1) = \phi(p(1),1)$ and $\sigma_B(1) = \sigma_B(0)$ otherwise, $\sigma_B(0) = \frac{\delta p(1)}{\delta+c+\varepsilon(1+\delta)}$.

The candidate equilibrium constitutes PBE of the perturbed game under some parameter restrictions. Below I consider each player and state separately:

Buyers in state $I = 1$: Note first that $\mu(1) = \frac{2\mu}{\mu+1}$ follows directly from Bayes rule. When observing $p(1) = \mu(1)$, the buyer is indifferent and so $\sigma_B(1) = 1$ constitutes a best response. Next, if observing $p(1) \neq \mu(1)$, $\phi(p(1),1) = 0$. Then $\sigma_B(1) = \sigma_B(0) = \frac{\delta p(1)}{\delta+c+\varepsilon(1+\delta)}$ constitutes a best response only if the strategy of the seller, along with buyer beliefs, are such that buyers are indeed indifferent.[45] If $p(1) = 1$, then buyer beliefs about z imply that high quality is a best response for sellers. As such buyers are indifferent and $\sigma_B(1) = \sigma_B(0)$ constitutes a best response. Next, if $p(1) \in [0,\mu(1)) \cup (\mu(1),1)$, buyers believe with certainty that $z = 1$. As $z = 1$ imply that the seller is indifferent for $\sigma_B(1) = \sigma_B(0)$, $\sigma_S(1) = p(1)$ would constitute a best response of the seller, which again must imply that $\sigma_B(1) = \sigma_B(0)$ is a best response of the buyer.

The seller in state $I = 1$: I first verify that, following an equilibrium price proposal, $\sigma_S(1) = 0$ is a best response for any z. Note first that if low quality is a best response for $z = -1$, then low quality must be a best response for all z. Thus, low quality in state $I = 1$ is best response for all realizations of z if

$$\delta(V_S^*(1,\boldsymbol{\sigma}_B,\boldsymbol{p}) - V_S^*(0,\boldsymbol{\sigma}_B,\boldsymbol{p})) \leq c - \varepsilon.$$

Given the mix played by the buyer in state $I = 0$, the the seller is exactly indifferent in state $I = 0$ if $z = 1$. This implies that $\delta(V_S^*(1,\boldsymbol{\sigma}_B,\boldsymbol{p}) - V_S^*(0,\boldsymbol{\sigma}_B,\boldsymbol{p})) = \sigma_B(0)(c+\varepsilon)$. Using this

[45] $\sigma_B(0) = \frac{\delta p(1)}{\delta+c+\varepsilon(1+\delta)}$ is derived from $\delta(V_S^*(1,\boldsymbol{\sigma}_B,\boldsymbol{p}) - V_S^*(0,\boldsymbol{\sigma}_B,\boldsymbol{p})) = \sigma_B(0)(c+\varepsilon)$, where $V_S^*(1,\boldsymbol{\sigma}_B,\boldsymbol{p})$ are substituted out under the assumption of equilibrium play. Note that $V_S^*(1,\boldsymbol{\sigma}_B,\boldsymbol{p})$ are taken to mean *expected* continuation payoffs with respects to z.

property, and replacing $\sigma_B(0)$, the condition above can be replaced with

$$\frac{\delta p(1)(c+\varepsilon)}{\delta + c + \varepsilon(1+\delta)} \leq c - \varepsilon.$$

The LHS is increasing in ε while the RHS is decreasing in ε. Furthermore, the condition clearly holds with a strict inequality for $\varepsilon = 0$. Thus, there is some $\overline{\varepsilon} > 0$ such that the condition holds for all $\varepsilon \leq \overline{\varepsilon}$.

Next, I verify that $p(1) = \mu(1)$ is a best response. Note that among the possible deviations from $p(1) = \mu(1)$, $p(1) = 1$ must the the optimal one. Also, the gain from deviating will be decreasing in z. As such, if it is not optimal to deviate from $p(1) = \mu(1)$ for $z = -1$ it is never optimal to deviate. Thus, conditional on play following a deviation in the posted price, $p(1) = \mu(1)$ is a best response if

$$p(1) + \delta V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p}) \geq \sigma_B(0)(1 - c + \varepsilon) + V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}).$$

Re-arranging and using the property that $\delta(V_S^*(1, \boldsymbol{\sigma}_B, \boldsymbol{p}) - V_S^*(0, \boldsymbol{\sigma}_B, \boldsymbol{p})) = \sigma_B(0)(c+\varepsilon)$ along with $\sigma_B(0) = \frac{\delta p(1)}{\delta + c + \varepsilon(1+\delta)}$, the condition simplifies to

$$c \geq \varepsilon(\delta - 1)$$

which holds for all $\varepsilon \geq 0$.

Following off-path price proposals, the seller will play as if in state $I = 0$. I check that these strategies are consistent with an equilibrium below.

Buyers in state $\boldsymbol{I = 0}$: Note first that if $p(0) = 1$, then buyer beliefs about z imply that high quality is a best response for sellers. As such buyers are indifferent and $\sigma_B(0)$ constitutes a best response. Next, if $p(0) < 1$, buyers believe with certainty that $z = 1$. As $z = 1$ imply that the seller is indifferent given $\sigma_B(0)$, $\sigma_S(0) = p(0)$ would constitute a best response of the seller, which again must imply that $\sigma_B(0)$ is a best response of the buyer.

The seller in state $\boldsymbol{I = 0}$: First, since $\sigma_B(0)$ is independent of $p(0)$, $p(0) = 1$, must constitute a best response. Furthermore, given $\sigma_B(0)$, high quality is a best response for all realizations of z. Finally, for $z = 1$, the seller is indifferent. Thus, for $p(0) < 1$ and $z = 1$, $\sigma_S(0) = p(0)$ constitutes a best response, and so buyer beliefs about the realization of z and seller play following an off-path proposal are correct.

In conclusion, by the steps above the candidate equilibrium constitutes an equilibrium of the perturbed game.

Finally, note that along the equilibrium path, the strategy of the seller is identical to that in Proposition 1 and that only the buyer strategies depend on ε. Letting $\varepsilon \to 0$, $\sigma_B(0) = \frac{\delta p(1)}{\delta+c+\varepsilon(1+\delta)}$ converges to $\frac{\delta p(1)}{\delta+c}$, which is equal to the strategy of buyers in state $I=0$ in the unperturbed game. Thus, the equilibrium characterized in Proposition 1 is purifiable in case of one-period memory.

D.2 A purifiable equilibrium with buyer-posted prices

The equilibrium characterized in Proposition 2 is not purifiable. In the equilibrium of Proposition 2, the seller conditions quality choice on price, which is what facilitates reputational concerns. However, having the seller condition on price requires that the seller is indifferent between high and low quality, but in the perturbed version of the game the seller cannot be indifferent.

While the equilibrium characterized in Proposition 2 is not purifiable, there are purifiable equilibria with buyer-posted prices. Consider, e.g., the following equilibrium candidate of the game described in section 2: Buyers post a price of zero and purchase with certainty in all states; the seller chooses low quality on both states regardless of posted price. This clearly constitutes an equilibrium as no player can profitably deviate. Furthermore, it is straight forward to construct an identical equilibrium of the perturbed version of the game.